# On the Energy Efficiency of Wireless Sensor Networks with Local Fusion Centers

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***Abstract*—In general, a wireless sensor network (WSN) consists of multiple sensors and one fusion center (FC). Each sensor can make a decision for given hypothesis testing and send its decision to the FC where decision fusion takes place. This centralized structure for WSNs can cause various problems. For example, if this FC is damaged or destroyed, the whole WSN collapses. Therefore, WSNs can be more robust if multiple local FCs (LFCs) can be deployed. In this paper, we study the energy efficiency of WSNs with LFCs, which will reveal another advantage of the use of LFCs, and determine the optimal number of LFCs under certain conditions.**

**Keywords**: Wireless sensor networks, energy efficiency, local fusion centers

## I. Introduction

For environmental and industrial monitoring and surveillance purpose, wireless sensor networks (WSNs) are widely used in various applications [1]. For a given sensing area, sensors are distributed and detect an unknown target or event locally. Each sensor makes a decision with its own observations and sends its local decision to a fusion center (FC). This process can be seen as distributed detection [2] [3] in certain applications. To see the performance of distributed detection when the number of sensors increases, asymptotic analysis is considered in [4]. For independent and identically distributed (iid) observations at each sensor, it is shown that the error probability of decision fusion can decay exponentially to zero. For non-ideal channels, in [5], it is also shown that the error probability can approach zero by increasing the number of sensors under certain conditions.

While the performance of distributed detection can be improved as the number of sensors increases, the increase of cost is not relatively well addressed. Clearly, the total transmission power within a WSN increases with as the number of sensors. This implies that the performance improvement results from higher energy consumption for transmitting sensors' decisions to the FC. To reduce the energy consumption, in this paper, we consider the use of multiple local FCs (LFCs). Sensors transmit their decisions to any nearest LFC and LFCs are connected to share the local decision information collected from sensors. Due to the presence of multiple LFCs, the resulting WSN can be robust against the failure of some LFCs (which could happen under attack or running out its energy source).

In order to study the energy efficiency of WSNs with LFCs, we consider a simple model in this paper. Under certain conditions, the total transmission energy is to be found as a closed-form expression.

Note that we focus on the energy efficiency when distributed detection [2] is considered for WSNs in this paper. While this setup simplifies the analysis for energy efficiency, we do not take into account any networking issues which are becoming important [6].

## II. System Models

In this section, we consider two different types of WSNs for comparison. The first type of WSNs is the conventional one consisting of multiple sensors and one FC, while A WSN of the second type consists of multiple sensors and multiple LFCs. In Fig. 1, the two different types of WSNs are illustrated.

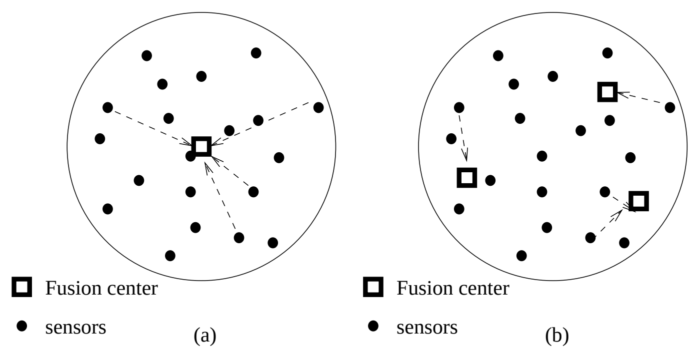


Fig. 1. WSNs: (a) the first type where there is one FC; (b) the second type where there are multiple LFCs.

Suppose that a WSN consists of $N$ sensors. Each sensor can perform a hypothesis testing for an event that can be observed uniformly over the sensing area. The decision results of sensors are to be sent back to FC or LFCs. The transmission of sensors' decisions to FC can be seen as a multiple access channel problem [7]. For efficient channel use, type-based transmissions [8] [9] can be used. Furthermore, energy efficient transmissions can be derived if the channel conditions are exploited in type-based transmissions [10]. However, in this paper, we only consider simple transmission strategies without any constraint on bandwidth.

In order to quantify the amount of the energy use to transmit the sensors' decisions, we consider the following assumptions for the first type:

A1) A whole sensing area is a circle of radius of $r$.
A2) Sensors are uniformly and independently distributed over the sensing area.
A3) The FC is located at the center of the sensing area.

In the second type, we have multiple LFCs and the assumption A3) is replaced as follows:

A3′) LFCs are uniformly and independently deployed.

In a WSN of the first type, all the sensors transmit their decisions to the FC, which is only one FC. On the other hand, in a WSN of the second type, sensors transmit their decisions to a nearest LFC. For convenience, we consider cells that cover a whole sensing area and assume that an LFC is located at the center of each cell. Denote by $M$ the number of LFCs or the number of cells. Once each LFC receives the decisions from the sensors in a cell, the LFCs are to exchange local information of sensors' decisions in a cell. Although there are various approaches to exchange local decisions, in this paper, for the sake of simplicity, we assume that each LFC can directly transmit its local decision to the other LFCs. Thus, there should be $M-1$ transmissions from one LFC. This direct transmission between LFCs is referred to as backhaul transmission in this paper. It is noteworthy that this direct backhaul transmission for local information exchange between LFCs is not efficient as it does not exploit the broadcasting nature of wireless communications and there should be more efficient approaches (in terms of transmission energy), which however, are not studied in this paper.

## III. Total Transmission Powers

### A. First Type

Taking the location of the FC as the origin, the location of a sensor can be characterized by the distance between the FC and sensor, $x \in (0, r]$, and reference angle $\theta \in (0, 2\pi]$. Then, the probability distribution function (pdf) of the sensor's location is given by

$$f(x,\theta) = \frac{1}{\pi r^2}, \;\; x \in (0,r], \;\; \theta \in (0, 2\pi]$$

according to A2).

If the distance between a transmitter and receiver is $y$, the received signal power is given by [11]

$$P_{rx} = \frac{\alpha P_{tx}}{y^\eta}, \tag{1}$$

where $\alpha$ is a certain constant that depends on the antenna gains, but independent of $y$, $P_{tx}$ is the transmission power, and $\eta$ is the path loss exponent. Suppose that the transmission power from any sensor is decided to keep the received signal power constant. Denote by $\bar{P}_{rx}$ the required received signal power. Then, from (1), for a sensor with $x$ distance, the transmission power is given by

$$P_{tx}(x) = \frac{\bar{P}_{rx}}{\alpha} x^\eta. \tag{2}$$

The total average transmission power from $N$ sensors uniformly distributed is given by

$$\begin{aligned} P_{total,I} &= \int_0^r P_{tx}(x) f_N(x) \mathrm{d}x \\ &= N \int_0^r P_{tx}(x) \frac{2x}{r^2} \mathrm{d}x, \end{aligned} \tag{3}$$

where $f_N(x)\mathrm{d}x$ denotes the average number of sensors on the intersection of the two circles of radii $x + \mathrm{d}x$ and $x$:

$$\begin{aligned} f_N(x)\mathrm{d}x &= N \left( \int_0^{x+\mathrm{d}x} \int_0^{2\pi} f(x,\theta) d\theta - \int_0^{x} \int_0^{2\pi} f(x,\theta) d\theta \right) \\ &= N \frac{2x}{r^2} \mathrm{d}x. \end{aligned}$$

Substituting (2) into (3), we have

$$P_{total,I} = N B(\bar{P}_{rx}, r), \tag{4}$$

where $B(P, r)$ is the normalized total transmission power when the coverage area is a circle of radius $r$ and $P$ is the target received signal power, which is given by

$$B(P,r) = \frac{2P}{\alpha(\eta+2)} r^\eta.$$

The total average transmission power increases linearly with $N$ and more rapidly with $r$.

### B. Second Type

In order to find the total transmission power in a WSN of the second type, we need to consider the powers for two different transmissions: *i*) the transmission power within a cell; *ii*) the backhaul transmission power between LFCs.

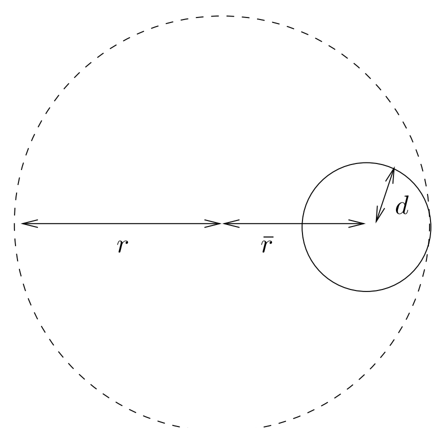


Fig. 2. The whole sensing area that is represented by a circle of dashed line and a sensing area of an LFC that is represented by a circle of solid line.

In Fig. 2, a geographical model with uniformly distributed $M$ LFCs is shown. The average number of sensors within a cell is $K = \frac{N}{M}$. Furthermore, the average radius of a cell is given by

$$d = \frac{r}{\sqrt{M}}.$$

Note that a cell is shown in Fig. 2 by a circle of solid line. The total transmission power transmitted by sensors within a cell is given by

$$P_{cell,II} = KB(\bar{P}_{rx}, d). \tag{5}$$

Since there are $M$ cells in the whole sensing area, the total transmission power transmitted by sensors in a WSN of the second type is given by

$$P_{sensor,II} = MP_{cell,II} = NB(\bar{P}_{rx}, d)., \tag{6}$$

We now need to find the the backhaul transmission power between LFCs. According to A3′), the distance between any two different LFCs can be considered as the distance between two random points in a circle of radius $\bar{r} = r - d$, which is denoted by $W$. Let $f(w)$ denote the distribution of the distance between any two different LFCs, where $0 \le w \le 2\bar{r}$. Then, the backhaul transmission power from an LFC is given by

$$\begin{aligned} P_{lfc,II} &= (M-1)\int_0^{2\bar{r}} P_{tx}(w) f(w)\mathrm{d}w \\ &= (M-1)\mathbb{E}[P_{tx}(W)] \\ &= (M-1)\frac{\bar{P}_{rx}}{\alpha}\mathbb{E}[W^\eta]. \end{aligned} \tag{7}$$

Here, we assume that the target received signal power for LFC backhaul communications is also $\bar{P}_{rx}$. Since there are $M$ LFCs, the total backhaul transmission power becomes

$$\begin{aligned} P_{bh,II} &= MP_{lfc,II} \\ &= M(M-1)\frac{\bar{P}_{rx}}{\alpha}\mathbb{E}[W^\eta]. \end{aligned} \tag{8}$$

The pdf of $W$ can be found in [12] and the moment of $W$ is given by [13, Eq. (2.3.68), p.207]

$$\mathbb{E}[W^\eta] = G_\eta \bar{r}^\eta \tag{9}$$

where

$$G_\eta = \frac{2^{\eta+4}}{\sqrt{\pi}} \frac{1}{(\eta+2)(\eta+4)} \frac{\Gamma\left(\frac{\eta+3}{2}\right)}{\Gamma\left(\frac{\eta}{2}+2\right)}.$$

## IV. Energy Consumption and Optimal Number of LFCs

In this section, we attempt to find the total average energy consumption to transmit all the sensors' decision within the whole sensing area.

Suppose each sensor requires $\tau$ seconds to transmit its decision to a FC or LFC. In a WSN of the first type, from (4), the total average energy consumption becomes

$$\begin{aligned} E_{total,I} &= \tau P_{total,I} \\ &= \tau NB(\bar{P}_{rx}, r). \end{aligned} \tag{10}$$

In a WSN of the second type, from (6), the energy consumption to transmit sensors' decisions is given by

$$E_{sensor,II} = \tau NB(\bar{P}_{rx}, d). \tag{11}$$

At each LFC, the information from $K$ sensors in a cell is received. Thus, without any compression, the transmission time to send this information to another LFC will be $\tau_{bh} = K\tau$. If any compression is considered, this transmission time could be shorter than $K\tau$. For convenience, define the LFC compression ratio (LCR) as

$$\beta = \frac{\tau_{bh}}{K\tau} \le 1.$$

We consider two different LCR values:

$$\beta = \begin{cases} \beta_{nc} = 1, & \text{if no compression is used;} \\ \beta_c = \frac{1}{K}, & \text{if compression is used.} \end{cases} \tag{12}$$

From (8), the total average energy consumption for the backhaul transmission is given by

$$E_{bh,II} = \tau_{bh} M(M-1)\frac{\bar{P}_{rx}}{\alpha}\mathbb{E}[W^\eta]. \tag{13}$$

Finally, the total average energy consumption for a WSN of the second type is

$$\begin{aligned} E_{total,II} &= E_{sensor,II} + E_{bh,II} \\ &= \tau NB(\bar{P}_{rx}, d) + \tau_{bh} M(M-1)\frac{\bar{P}_{rx}}{\alpha}\mathbb{E}[W^\eta]. \end{aligned} \tag{14}$$

Define the energy ratio as

$$C = \frac{E_{total,II}}{E_{total,I}}, \tag{15}$$

which can show the energy efficiency of the second type WSN over the first type WSN. If $C$ is less than 1, we can see that the use of LFCs can save the energy consumption compared with the conventional WSN, i.e., a WSN of the first type. Substituting (10) and (14) into (15), we have

$$C = \left(\frac{d}{r}\right)^\eta + \frac{\beta(\eta+2)G_\eta}{2}\left(\left(\frac{r}{d}\right)^2 - 1\right)\left(1 - \frac{d}{r}\right)^\eta. \tag{16}$$

For convenience, let $A_\eta = \frac{\eta+2}{2}G_\eta$ and $\rho = \frac{d}{r}$. Then, depending on the LFC compression, we have two different energy ratios:

$$C_{nc}(\rho) = \rho^\eta + A_\eta\left(\frac{1}{\rho^2} - 1\right)(1-\rho)^\eta; \tag{17}$$

$$C_c(\rho, N) = \rho^\eta + \frac{A_\eta}{N}\left(\frac{1}{\rho^2}\left(\frac{1}{\rho^2} - 1\right)\right)(1-\rho)^\eta. \tag{18}$$

The energy ratio without LFC compression, $C_{nc}(\rho)$, becomes a function of $\rho$ and independent of the number of sensors, $N$. On the other hand, the energy ratio with LFC compression, $C_c(\rho, N)$, is a function of both $\rho$ and $N$.

If the LFC compression is not employed, the energy efficiency of the second type WSN may not be high and even could be worse than that of the first type WSN. From (17), if $\rho$ approaches 0, we can see that $C_{nc}(\rho)$ approaches infinity. In other words, the second type WSN becomes significantly inefficient as the number of LFCs approaches infinity. This inefficiency results from heavy backhaul overhead. However, there should be an optimal value of $\rho$ that minimizes the energy ratio. First, we consider (17). In order to find the minimum of $C_{nc}(\rho)$, we can take the derivative with respect to $\rho$:

$$\frac{\mathrm{d}C_{nc}(\rho)}{\mathrm{d}\rho} = \eta\rho^{\eta-1} - A_\eta \left( \frac{2(1-\rho)^\eta}{\rho^3} - \frac{\eta(1-\rho^2)(1-\rho)^{\eta-1}}{\rho^2} \right).$$

If $\eta > 1$, we can easily see that the first term is increasing from 0 to $\eta$ as $\rho$ increases from 0 to 1, while the term in the parentheses of the second term is decreasing from $\infty$ to 0. Therefore, there exists a unique minimum. Although a closed-form expression for the the optimal $\rho$ that minimizes $C_{nc}(\rho)$ is not available, it could be easily found by using a numerical technique.

We can have a similar observation with $C_c(\rho, N)$ in (18). There is an optimal $\rho \in (0, 1)$ that minimizes $C_c(\rho, N)$ for a given $N$. For convenience, denote by $\rho^*_{nc}$ and $\rho^*_c$ the optimal $\rho$'s that minimize $C_{nc}(\rho)$ and $C_c(\rho, N)$, respectively. Since the number of LFCs is $M = \frac{1}{\rho^2}$, once the optimal $\rho$ is found, we can determine the optimal number of LFCs. For example, if $\rho^* = 0.5$, then 4 LFCs are required to minimize the energy to send sensors' decisions to all the LFCs.

## V. Numerical Results and Discussion

In order to see the energy efficiency, we find the energy ratios in (17) and (18) for various values of $\rho$ when the path loss exponent, $\eta$, is 3, and show the result in Fig. 3. Since the second type WSN becomes the first type as $\rho \to 1$ (i.e., single LFC), it is clear that $C_{nc}(1) = C_c(1, N) = 1$ as shown in Fig. 3. Furthermore, as $\rho \to 0$, there is a significant energy consumption for the backhaul transmission (due to a large number of LFCs), which results in a poor energy efficiency of the second type WSN. Thus, we can expect that $\lim_{\rho\to 0} C_{nc}(\rho) = \lim_{\rho\to 0} C_c(\rho, N) = \infty$ as shown in Fig. 3. However, we are interested in the minimum values of $C_{nc}(\rho)$ and $C_c(\rho, N)$. As discussed in Section IV, both $C_{nc}(\rho)$ and $C_c(\rho, N)$ have a minimum point as shown in Fig. 3, and the minimum values are less than 1, which show that the second type WSN can be energy efficient once the number of uniformly deployed LFCs is optimal.

Fig. 4 shows the optimal $\rho^*$ for different values of the path loss exponent, $\eta$, when $N = 100$. It is shown that the energy ratio decreases with $\eta$. Since the propagation loss becomes more significant as $\eta$ increases, it is essential to reduce the transmission distance to reduce the transmission power and energy. Since the second type WSN can efficiently reduce the transmission distance, it becomes more energy efficient as $\eta$ increases. From Fig. 4 (a), we can see that the optimal value of $\rho$ is 0.5 for the second type of WSN when the LFC compression is used and $\eta = 3$, which means that 4 LFCs are optimal (i.e., $M = 4$) for the second type WSN in this case. Furthermore, as shown in Fig. 4 (b), the second type of WSN requires less than 20% energy of the first type of WSN.

Fig. 5 shows the energy ratio when the second type WSN uses LFC compression. As $N$ increases, the energy ratio decreases and the second type WSN becomes more energy efficient. Due to the LFC compression, the energy consumption for backhaul transmissions becomes negligible compared to that for sensors' transmissions. Thus, the more sensors, the better energy efficiency the second type WSN can achieve.

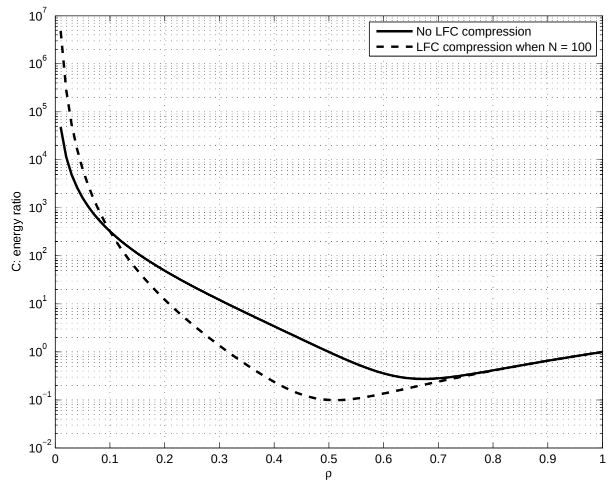


Fig. 3. Energy ratios (with and without LFC compression) versus $\rho$.

## VI. Concluding Remarks

In this paper, we studied the energy efficiency of WSNs with LFCs for signal transmissions. Under certain conditions, it was shown that WSNs with LFCs can be more energy efficient than WSN with single FC. A closed-form expression for the energy efficiency was derived and used to find an optimal number of LFCs. It was found that the optimal number of LFCs is independent of the sensing area and, without LFC compression, it is also independent of the number of sensors. On the other hand, if LFC compression is employed, the optimal number of LFCs increases with the number of sensors. These observations have been encouraging the use of multiple LFCs for WSNs.

While an inefficient but simple backhaul transmission approach was considered to see the energy efficiency of WSNs with LFCs in this paper, the energy efficiency can be further improved if efficient backhaul transmission approaches are used, which will be investigated in the future.

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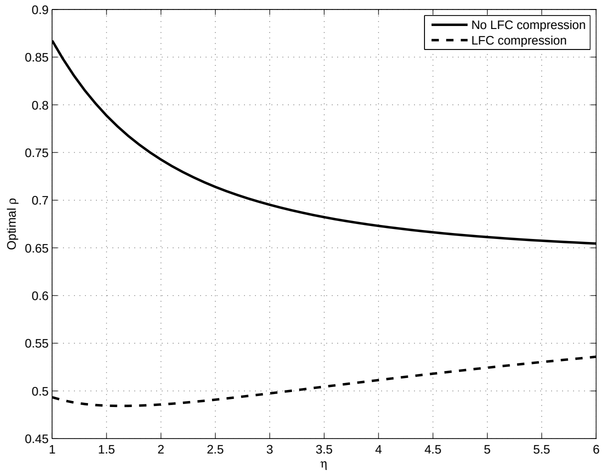
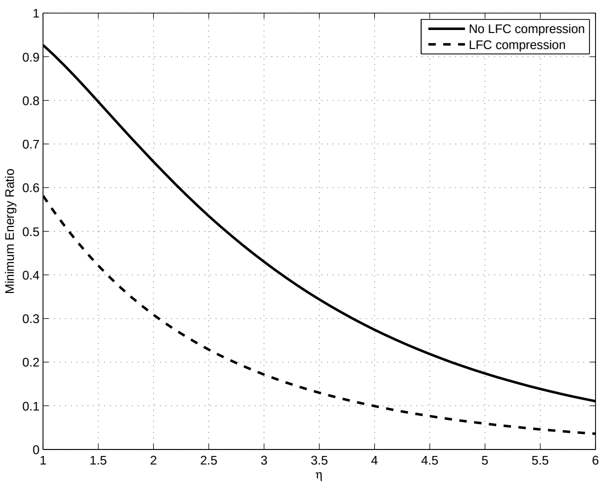
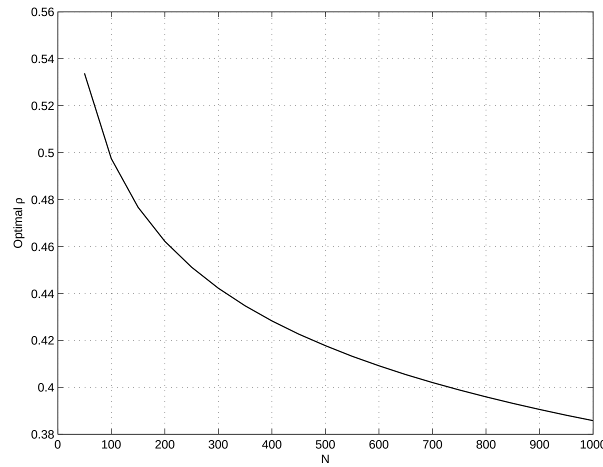



(a)

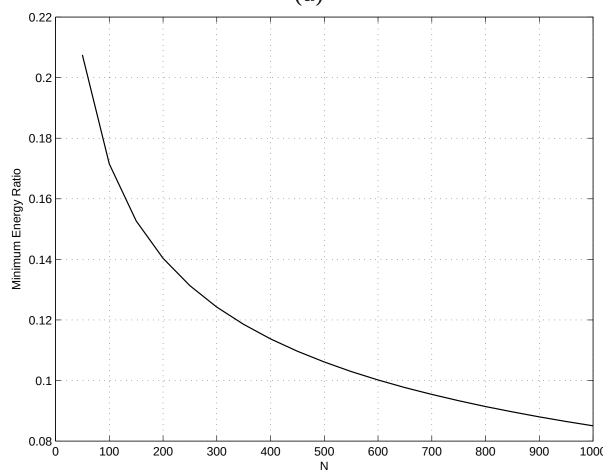



(b)

Fig. 4. Energy ratios (with and without LFC compression) versus $\eta$.




(a)




(b)

Fig. 5. Energy ratio (with LFC compression) versus $N$.

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