Truncated unscented particle filter for dealing with non-linear inequality constraints

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***Abstract*—This paper addresses state estimation where domain knowledge is represented by non-linear inequality constraints. To cope with non-Gaussian state distribution caused by the utilisation of domain knowledge, a truncated unscented particle filter method is proposed in this paper. Different from other particle filtering schemes, a truncated unscented Kalman filter is adopted as the importance function for sampling new particles in the proposed truncated unscented particle scheme. Consequently more effective particles are generated and a better state estimation result is then obtained. The advantages of the proposed truncated unscented particle filter algorithm over the state-of-the-art particle filters are demonstrated through Monte-Carlo simulations.**

I. Introduction

State estimation of dynamic stochastic systems is important and receives wide attention in different fields, such as automatic control, signal processing, communication systems, econometrics and so on. The goal of state estimation is to find an estimate of a state using a set of measurements. The dynamics of the state and the relation between the state and measurement are described by a state model and measurement model respectively.

The state could be estimated from Bayesian inference, from which the conditional probability density function (pdf) of the state conditioned by the measurement is obtained. After deriving the conditional pdf of the state vector by Bayesian inference, the minimum mean square error (MMSE) or maximum a posterior (MAP) estimators [1] could be calculated as the state estimation. As mentioned in [1], if the state and measurement models are both linear and Gaussian, the famous Kalman filter could be applied to obtain an exact solution of the conditional pdf. However, for most real life applications, the linear and Gaussian assumptions do not hold. Different types of algorithms are applied in order to deal with such non-linear and non-Gaussian models. The classical ones include the variations of the standard Kalman filter (including extended Kalman (EKF) or unscented Kalman filter (UKF)), and particle filter [2].

In some real state estimation problems, the state vector values in stochastic dynamic systems are restricted to a sub-area of the state space. This is usually the consequence of some physical restrictions or elicited qualitative knowledge about the systems of interest. For instance, when a vehicle moves on the road, its position is constrained to be within the boundaries of the road and its speed is also generally within the corresponding speed limits specified by the Rule of the Road. Intuitively, because the constraints reduce the variability of the state vector, we can incorporate the constraint-related information into state estimation to achieve a more accurate result.

State estimation with state constraints is in general challenging and has attracted a considerable interest. Many approaches have been developed to deal with linear and/or equality constraints, which incorporate these constraints into the Kalman filtering framework. The standard ones include re-parameterizing and pseudo-measurement approaches [3] and [4], the optimization approach [5] and truncation approaches [6], [7] and [8]. However, the constraints may be both non-linear and inequality for some applications. Moreover, the state vector distribution is highly non-Gaussian due to the introduction of these non-linear constraints, and the Kalman filtering based methods are then not applicable. In order to more efficiently incorporate non-linear inequality constraints, the particle filtering approach is applied and modified to cope with constraints.

Lang et al. in [9] developed a simple and straightforward algorithm using the acceptance-rejection method to deal with nonlinear inequality constraints where only the particles within the constraint region are retained in particle filtering implementation. Although simple and straightforward, this method is inefficient and the probability of a particle satisfying the constraint may be very low if the constraints are quite restrictive. Furthermore, even if a sample is within the constraint region, it is more likely to be an outlier which has a lower measurement likelihood value. Shao et al. in [10] proposed a novel approach consisting of two stages: First, a set of particle candidates is drawn without consideration of the state constraints, while the candidates that do not satisfy the constraints are projected into the feasible area using a series of optimizations n the stage two. The limitation of this method is that by applying optimizations to force the particle to be within the feasible regions, the resulting particles are no longer representative samples of the posterior distribution of the state vector. So that this method is incorrect from a statistical point of view.

In this work, we propose an elegant truncated unscented particle filtering approach, which can deal with non-linear inequality constraints effectively. The proposed truncated unscented particle filter is based on the sequential importance sampling (SIS) method in [2]. It applies a truncated version of the UKF as the importance function to generate new particles. In this way, both the measurement and constraint information are considered in the sampling procedure, and more effective

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samples are then generated for state estimation. One method that is similar to our truncated unscented particle filter is proposed in [11]; however, this method only considers the first two moments of the posterior distribution. This Gaussian approximation may lead to less accurate state estimation.

The outline of this paper is as follows: Section II briefly describes the general problem of state estimation with constraints. The concept of truncated UKF is proposed in Section III, and Section IV presents the proposed truncated UPF idea for state estimation. The simulation results are presented in Section V, which show that our proposed method achieves a better performance than the current state-of-the-art ones.

II. State estimation with constraints

A stochastic dynamic system with state and measurement models described is described by

$$\begin{aligned} \mathbf{x}_{k+1} &= f_k(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k \\ \mathbf{z}_k &= h_k(\mathbf{x}_k) + \mathbf{v}_k \end{aligned} \tag{1}$$

where $\mathbf{x}_k \in R^{n_x}$ represents the state vector, $\mathbf{u}_k \in R^{n_u}$ represents the control vector, and $\mathbf{z}_k$ is the measurement vector; $f_k(\cdot,\cdot)$ and $h_k(\cdot)$ are linear/non-linear functions describing state/measurement models; and $\mathbf{w}_k$ and $\mathbf{v}_k$ represent the state and measurement noises, respectively, which can be described by the corresponding pdfs $p(\mathbf{w}_k)$ and $p(\mathbf{v}_k)$. In this work, $\mathbf{w}_k$ and $\mathbf{v}_k$ are assumed to have Gaussian distributions with $p(\mathbf{w}_k) = N(\mathbf{0}, Q_k)$ and $p(\mathbf{v}_k) = N(\mathbf{0}, R_k)$.

The aim of the state estimation is to estimate the state vector $\mathbf{x}_k$ of a stochastic dynamic system described in (1) based on the measurement $\mathbf{z}_k$. From the conditional pdf $p(\mathbf{x}_k|\mathbf{z}_k)$ derived from Kalman/particle filtering schemes, the minimum mean square error (MMSE) or maximum a posterior (MAP) estimators for $\mathbf{x}_k$ could be obtained.

In some applications, more information might be available to refine the distribution of the state vector $\mathbf{x}_k$, such as the physical condition which imposes a valid sub-space for the state vector $\mathbf{x}_k$. As highlighted in [11], this type of information can be represented as a general nonlinear inequality form in many scenarios

$$\mathbf{a}_k \leq \mathbf{C}_k(\mathbf{x}_k) \leq \mathbf{b}_k \tag{2}$$

where $\mathbf{C}_k$ is a mapping function: $R^{n_x} \rightarrow R^{n_c}$ and $\mathbf{a}_k, \mathbf{b}_k \in R^{n_c}$.

After introducing the constraint in (2), the conditional pdf $p(\mathbf{x}_k|\mathbf{z}_k)$ is modified to be $p_C(\mathbf{x}_k|\mathbf{z}_k)$ as:

$$p_C(\mathbf{x}_k|\mathbf{z}_k) \propto \begin{cases} p(\mathbf{x}_k|\mathbf{z}_k) & \text{if}\, \mathbf{x}_k \in \mathcal{C}_k \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $\mathcal{C}_k$ is the feasible region defined as:

$$\mathcal{C}_k = \{\mathbf{x}_k :, \mathbf{x}_k \in R^{n_x}, \mathbf{a}_k \leq \mathbf{C}_k(\mathbf{x}_k) \leq \mathbf{b}_k\} \tag{4}$$

The conditional density function $p_C(\mathbf{x}_k|\mathbf{z}_k)$ could be regarded as a truncation of $p(\mathbf{x}_k|\mathbf{z}_k)$ by the feasible region $\mathcal{C}_k$. It incorporates the constraint information by setting the probability values outside the feasible region to be zero. With the aid of the truncated conditional pdf $p_C(\mathbf{x}_k|\mathbf{z}_k)$, the uncertainty of the state vector $\mathbf{x}_k$ is then reduced and a more accurate state estimation is obtained.

III. Truncated unscented Kalman filter

As mentioned in [12], the truncated UKF is a filtering method which can be applied to estimate the conditional pdf $p_C(\mathbf{x}_k|\mathbf{z}_k)$ considering the constraint information. Truncated UKF is divided into two steps: the first step is to apply the traditional UKF method to estimate the mean and covariance matrix for approximating the conditional pdf $p(\mathbf{x}_k|\mathbf{z}_k)$, and the second step is to estimate the Gaussian approximated truncated probability $p_C(\mathbf{x}_k|\mathbf{z}_k)$ from $p(\mathbf{x}_k|\mathbf{z}_k)$ by incorporating the constraint information.

A. Unscented Kalman filter

If the state and measurement models in (1) are linear and Gaussian, the Kalman filter could be applied to calculate the conditional pdf and obtain the state estimation. However, in many situations the linear and Gaussian assumptions do not hold and some variations of the Kalman filter are proposed to deal with the non-linear and non-Gaussian models. One popular variation is the UKF. The UKF is based on the unscented transform (UT), which computes the first two moments of $p(\mathbf{x}_k|\mathbf{z}_k)$ using a set of $\sigma-$points. Compared with the extended Kalman filter (EKF), it obtains a better estimation if the non-linearities in state/measurement models are high.

Initially, we have the conditional pdf $p(\mathbf{x}_{k-1}|\mathbf{z}_{k-1})$ with the mean $\hat{\mathbf{x}}_{k-1|k-1}$ and covariance matrix $P_{k-1|k-1}$ at time $t-1$, a set of $\sigma-$points $\{\chi_{i,k-1|k-1}\}$ and corresponding weights $\{\omega_{i,k-1|k-1}\}$ could be calculated as:

$$\chi_{0,k-1|k-1} = \hat{\mathbf{x}}_{k-1|k-1}, \quad \omega_{0,k-1|k-1} = \frac{\kappa}{n_\chi + \kappa} \tag{5}$$

$$\begin{aligned} \chi_{i,k-1|k-1} &= \hat{\mathbf{x}}_{k-1|k-1} + (\sqrt{(n_\chi + \kappa) P_{k-1|k-1}})_i \\ \omega_{i,k-1|k-1} &= \frac{1}{2(n_\chi + \kappa)} \end{aligned} \tag{6}$$

$$\begin{aligned} \chi_{n_\chi+i,k-1|k-1} &= \hat{\mathbf{x}}_{k-1|k-1} - (\sqrt{(n_\chi + \kappa) P_{k-1|k-1}})_i \\ \omega_{n_\chi+i,k-1|k-1} &= \omega_{i,k-1|k-1} \end{aligned} \tag{7}$$

where n_χ and κ are preset parameters, $i = 1, ..., n_\chi$, and in total there are $2n_\chi + 1$ $\sigma-$points. $(A)_i$ represents the i-th column of the matrix A.

With the aid of the $\sigma-$points $\{\chi_{i,k-1|k-1}\}_{i=0,\ldots,2n_\chi}$ and corresponding weights $\{\omega_{i,k-1|k-1}\}_{i=0,\ldots,2n_\chi}$, the mean $\hat{\mathbf{x}}_{k|k}$ and covariance matrix $P_{k|k}$ of $p(\mathbf{x}_k|\mathbf{z}_k)$ could be derived by the prediction and correction steps of the UKF algorithm:

Predictions:

The first two moments of $p(\mathbf{x}_k|\mathbf{z}_{k-1})$ could be predicted as:

$$\hat{\mathbf{x}}_{k|k-1} = E(\mathbf{x}_k|\mathbf{z}_{k-1}) \approx \sum_{i=0}^{2n_\chi} \omega_{i,k-1|k-1} \chi_{i,k|k-1} \tag{8}$$

$$\begin{aligned} P_{k|k-1} &= E((\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1})(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1})^T|\mathbf{z}_{k-1}) \\ &\approx \sum_{i=0}^{2n_\chi} \omega_{i,k-1|k-1} (\chi_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1})(\chi_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1})^T \\ &+ Q_{k-1} \end{aligned} \tag{9}$$

where $\chi_{i,k|k-1}$ for every i is predicted from the state model as $f(\chi_{i,k-1|k-1}, \mathbf{u}_k)$.

Corrections:

After receiving the measurement $\mathbf{z}_k$, the state prediction results could be updated as:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_{k|k}(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}) \tag{10}$$

$$P_{k|k} = P_{k|k-1} - K_{k|k} P_{z,k|k-1} K_{k|k}^T \tag{11}$$

where $K_{k|k} = P_{xz,k|k-1}(P_{z,k|k-1})^{-1}$ is the filter gain and we have the following definitions:

$$\hat{\mathbf{z}}_{k|k-1} \approx \sum_{i=0}^{2n_\chi} \omega_i \mathcal{Z}_{i,k|k-1} \tag{12}$$

$$P_{z,k|k-1} \approx \sum_{i=0}^{2n_\chi} \omega_i (\mathcal{Z}_{i,k|k-1} - \hat{\mathbf{z}}_{k|k-1})(\mathcal{Z}_{i,k|k-1} - \hat{\mathbf{z}}_{k|k-1})^T + R_k \tag{13}$$

$$P_{xz,k|k-1} \approx \sum_{i=0}^{2n_\chi} \omega_i (\chi_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1})(\mathcal{Z}_{i,k|k-1} - \hat{\mathbf{z}}_{k|k-1})^T \tag{14}$$

$$\mathcal{Z}_{i,k|k-1} = h(\chi_{i,k|k-1}) \tag{15}$$

In this way, the mean $\hat{\mathbf{x}}_{k|k}$ and covariance $P_{k|k}$ of $p(\mathbf{x}_k|\mathbf{z}_k)$ are updated from the time instance $k-1$. $p(\mathbf{x}_k|\mathbf{z}_k)$ could then be described by these first two moments as a Gaussian distribution denoted as $N(\mathbf{x}_k|, \hat{\mathbf{x}}_{k|k}, P_{k|k})$.

B. Importance sampling based truncated probability estimation

No constraint information is taken into account for the traditional UKF framework. When the feasible region $\mathcal{C}_k$ is considered, according to the definitions in (3), the truncated conditional pdf $p_C(\mathbf{x}_k|\mathbf{z}_k)$ could be calculated as:

$$p_C(\mathbf{x}_k|\mathbf{z}_k) \propto \begin{cases} \xi_k^{-1} p(\mathbf{x}_k|\mathbf{z}_k) & \text{if } \mathbf{x}_k \in \mathcal{C}_k \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where ξ_k is a normalizing constant calculated as:

$$\xi_k = \int_{\mathcal{C}_k} p(\mathbf{x}_k|\mathbf{z}_k) d\mathbf{x}_k \tag{17}$$

As mentioned in [11], a sampling based method could be applied to estimate the mean and covariance of the truncated conditional pdf $p_C(\mathbf{x}_k|\mathbf{z}_k)$, which can then be approximated by a Gaussian distribution. The sampling could be directly applied on $p(\mathbf{x}_k|\mathbf{z}_k)$ (approximated by $N(\mathbf{x}_k|, \hat{\mathbf{x}}_{k|k}, P_{k|k})$ from the UKF procedure) and the obtained samples within the constraint region $\mathcal{C}_k$ are kept while other samples are discarded; however, we notice that sometimes the probability of obtaining a valid sample is low. The mean and covariance thus can not be estimated accurately by a limited number of samples. In order to solve this problem, we refer to the importance sampling technique. The samples are not obtained directly from $p(\mathbf{x}_k|\mathbf{z}_k)$, but from another importance function $q(\mathbf{x}_k)$ whose volume should be largely within the constrained region $\mathcal{C}_k$. In this work, $q(\mathbf{x}_k)$ is chosen as a Gaussian distribution with the mean being the projection of $\hat{\mathbf{x}}_{k|k}$ (the mean estimated by the UKF scheme as in (10)) into the nearest point in the feasible region, and the covariance being $P_{k|k}$ which is the same as the one calculated from the UKF scheme.

From a sample set $\{\mathbf{x}_k^{c,i}\}_{i=1,\ldots,N}$ drawn from $q(\mathbf{x}_k)$ in the constraint region $\mathcal{C}_k$, the approximate mean $\hat{\mathbf{x}}_{k|k}^c$ and covariance $P_{k|k}^c$ of $p_C(\mathbf{x}_k|\mathbf{z}_k)$ could be estimated as:

$$\hat{\mathbf{x}}_{k|k}^c = \frac{1}{N} \sum_{i=0}^{N} \mathbf{x}_k^{c,i} w_k^{c,i} \tag{18}$$

$$P_{k|k}^c = \frac{1}{N} \sum_{i=0}^{N} (\mathbf{x}_k^{c,i} - \hat{\mathbf{x}}_{k|k}^c)(\mathbf{x}_k^{c,i} - \hat{\mathbf{x}}_{k|k}^c)^T w_k^{c,i} \tag{19}$$

where $\omega_k^{c,i} = N(\mathbf{x}_k^{c,i}|, \hat{\mathbf{x}}_{k|k}, P_{k|k})/q(\mathbf{x}_k^i)$ taking into account that $p(\mathbf{x}_k|\mathbf{z}_k)$ is approximated by $N(\mathbf{x}_k|, \hat{\mathbf{x}}_{k|k}, P_{k|k})$ from the UKF procedure.

By applying a standard unscented Kalman filter followed by an importance sampling based method for truncated conditional probability estimation, the truncated UKF scheme estimates the mean $\hat{\mathbf{x}}_{k|k}^c$ and covariance $P_{k|k}^c$ of $p_C(\mathbf{x}_k|\mathbf{z}_k)$, which is approximated by a Gaussian distribution $N_C(\mathbf{x}_k|, \hat{\mathbf{x}}_{k|k}^c, P_{k|k}^c)$. It inherits the advantages of the UKF for coping with highly nonlinear models and incorporates the constraint information in an efficient way.

IV. TRUNCATED UNSCENTED PARTICLE FILTER

Due to the fact that the distribution is truncated by the constraints, the $p_C(\mathbf{x}_k|\mathbf{z}_k)$ may not be accurately represented by a single Gaussian distribution as estimated from the truncated unscented Kalman filter. In order to represent the conditional pdf in a better way, the particle filtering scheme is applied.

The particle filtering scheme is rooted in Monte-Carlo sampling, which approximates a pdf $p_C(\mathbf{x}_k|\mathbf{z}_k)$ by a set of weights and particles $\{\mathbf{x}_k^i, \omega_k^i\}_{i=1,\ldots,N}$ as:

$$p_C(\mathbf{x}_k|\mathbf{z}_k) \approx \sum_{i=0}^{N} \omega_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) \tag{20}$$

The particle filtering scheme adopts a sequential importance sampling method to estimate $\{\mathbf{x}_k^i, \omega_k^i\}_{i=1,\ldots,N}$ from the weights and samples at $k-1$. An importance function $q(\mathbf{x}_k|\mathbf{x}_{k-1}^i, \mathbf{z}_k)$ is applied to generate the i-th particle $\mathbf{x}_k^i$ and the corresponding weight ω_i is then updated as in [2]. Unlike the generic particle filter in [2] which generates particles from the importance function solely determined by the state model, the unscented particle filtering (UPF) scheme proposed in [13] applies a UKF to estimate the importance function for generating each particle. The estimated importance function is a local approximation of the optimal importance function, given the assumption that the state and measurement models are both linear and Gaussian in a local region nearby each particular particle. By applying the UKF framework to estimate the importance function, the measurement information is incorporated into the particle sampling procedure and the generated particles are more likely from the region with high measurement likelihood.

However, for the traditional UPF, the constraint information is not taken into account. In order to make use of the constraint information, instead of the UKF, the truncated UKF discussed in Section III is applied to obtain the importance function and a corresponding truncated UPF scheme is derived. The procedure of the truncated UPF scheme for estimating $p_C(\mathbf{x}_k|\mathbf{z}_k)$ is outlined as Table I.

Here $p(\mathbf{z}_k|\mathbf{x}_k)$ and $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ in Table I represent the measurement likelihood function and the state transition function respectively, which are determined by the state and measurement models in (1). C_i is the normalization factor considering the constraint on $\mathbf{x}_k$, which is estimated as:

$$C_i = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}^i)\mathbf{C}_k(\mathbf{x}_k)d\mathbf{x}_k \tag{21}$$

and it can be estimated from the Monte-Carlo integration as [11], which is similar to the procedure of estimating the mean and covariance of the truncated distribution.

The truncated UPF scheme applies the truncated UKF as the importance function to take both the measurement and constraint information into account in the particle sampling procedure. In this way, a more accurate representation of the truncated conditional density function $p_C(\mathbf{x}_k|\mathbf{z}_k)$ can be obtained.

V. SIMULATIONS

A vehicle is simulated to move on a bending road segment. The boundaries of the road are defined by two arcs centered at the origin of a Cartesian coordinate system with radius of r_1 = 96m and r_2 = 100m, respectively. The vehicle dynamics are described by a state model driven by white noise acceleration:

$$\mathbf{x}_k = F \cdot \mathbf{x}_{k-1} + G \cdot \mathbf{w}_k \tag{22}$$

where

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix} \tag{23}$$

and $\mathbf{x}_k = [x_k \quad \dot{x}_k \quad y_k \quad \dot{y}_k]^T$. The variables (x_k, y_k) represent the position of the vehicle and $(\dot{x}_k, \dot{y}_k)$ represents the velocities. T is the sampling interval and assumed to be 1 second. $\mathbf{w}_k = (w_k^1, w_k^2)^T$ is a 2×1 vector representing the process noise. Each component of $\mathbf{w}_k$ follows a Gaussian distribution with zero mean and a standard deviation of 10. The road boundaries are considered as the state constraints, which are non-linear inequality described by

$$r_1 \leq \sqrt{x_k^2 + y_k^2} \leq r_2 \tag{24}$$

The vehicle is tracked by a range and bearing sensor modeled as:

$$\mathbf{z}_k = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \arctan(\frac{y_k}{x_k}) \end{bmatrix} + \mathbf{v}_t \tag{25}$$

where $\mathbf{v}_t$ is a Gaussian noise vector with mean $[0, 0]^T$ and covariance matrix $R = \begin{bmatrix} 5 & 0 \\ 0 & 0.001 \end{bmatrix}$.

A simulated trajectory based on the state model (22) and the corresponding measurements is plotted in Figure 1.

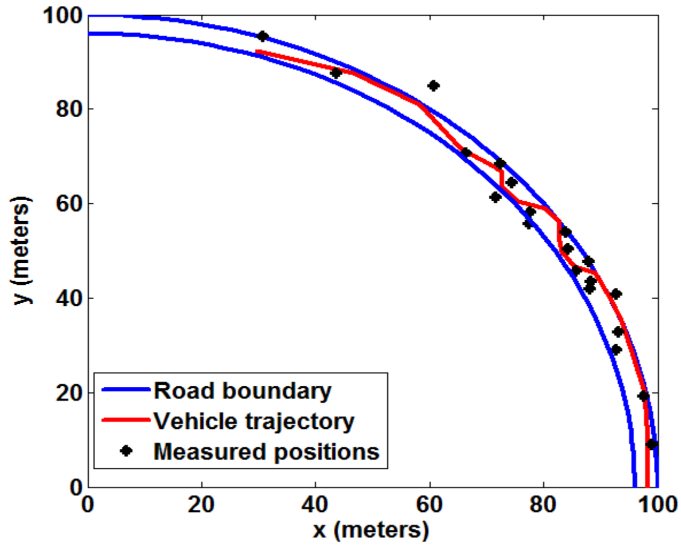

Fig. 1. The simulated trajectory of a vehicle moving on a bend road section and the measured positions.

The incorporation of the state constraint information in (24) could improve the state estimation performance and an example is presented to illustrate it. Figure 2 shows the comparison of the position estimation results by the truncated UPF proposed in this work and the original UPF without considering the constraint information. 100 particles are applied for each method. The mean square errors (MSEs) for these two methods are also calculated. It is evident in this figure that by incorporating the constraint information, the tracking result of truncated UPF is always within the road boundaries and less MSE is obtained.

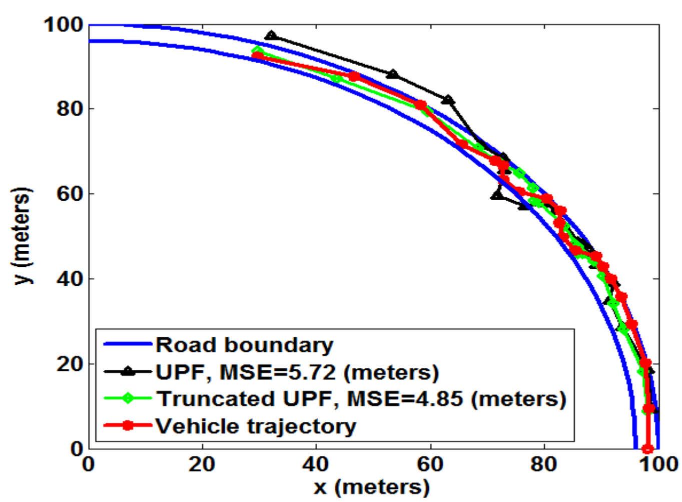

Fig. 2. The comparison results of the T-UPF and standard UPF. Better performance is achieved by T-UPF after incorporating the constraint information.

Next, the proposed method is compared with different methods that are able to cope with non-linear and inequality constraints. Methods include the acceptance-rejection method in [9], projection method in [10] and the method proposed in [11]. For a comprehensive analysis, 100 Monte-Carlo simulations are performed to generate the vehicle trajectories and

TABLE I. THE PROCEDURE OF THE TRUNCATED UPF.

Initially, we have a set of particles and weights $\{\mathbf{x}_{k-1}^i, \omega_{k-1}^i\}_{i=1,\dots,N}$ to approximate $p_C(\mathbf{x}_{k-1}|\mathbf{z}_{k-1})$. For each particle i, there is an associated Gaussian distribution $N(\mathbf{x}_{k-1}|\hat{\mathbf{x}}_{k-1}^i, P_{k-1}^i)$ estimated by the corresponding truncated UKF at time k-1.

I. Importance sampling:
For each particle i, the truncated UKF in Section III is applied to update $N(\mathbf{x}_{k-1}|\hat{\mathbf{x}}_{k-1}^i, P_{k-1}^i)$ at $k-1$ to a new distribution $N(\mathbf{x}_k|\hat{\mathbf{x}}_k^i, P_k^i)$, from which a new particle $\mathbf{x}_k^i$ is sampled.

II. Accept and rejection:
If the obtained sample $\mathbf{x}_k^i$ is within the constraint region in (2), the sample is accepted; otherwise, it is rejected.

III. Weight computing:
The weight corresponding to the accepted particle $\mathbf{x}_k^i$ is calculated as: $\omega_k^i \propto \omega_{k-1}^i \frac{p(\mathbf{z}_k|\mathbf{x}_k^i)p(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{C_i N(\mathbf{x}_k^i|\hat{\mathbf{x}}_k^i, P_k^i)}$

Finally, the weights are normalized to make $\sum_{i=1}^N \omega_k^i = 1$ and $p_C(\mathbf{x}_k|\mathbf{z}_k)$ is approximated by the new weights and particles as (20).

measurements. Each method with 100 particles is applied to obtain the position estimation results for Monte-Carlo simulations and the corresponding MSEs are calculated. The Mean and Standard deviation(Std) of the 100 MSEs are then estimated for comparison. The results are summarized in Table II,which shows that our method achieves the minimum Mean value of the MSEs as well as the smallest Standard deviation compared with other methods.

TABLE II. COMPARISONS OF DIFFERENT ALGORITHMS FOR INCORPORATING THE CONSTRAINT INFORMATION

	Accept rejection [9]	Projection [10]	Method in [11]	Proposed
Mean (meters)	10.89	6.92	6.76	5.40
Std (meters)	7.24	2.33	2.09	0.91

VI. CONCLUSIONS

In this work, we presented a truncated unscented particle filtering scheme to cope with non-linear and inequality constraints. Particle filtering was applied to deal with the non-Gaussian conditional pdf of the state vector due to the introduction of constraints. Unlike the traditional particle filtering methods, the proposed truncated unscented particle filtering scheme adopts the truncated UKF as the importance function from which particles were sampled. Both the measurement and constraint information were incorporated to obtain a better sampling scheme, and a more accurate state estimation result was thus obtained. From multiple Monte Carlo simulations, it was shown that our method achieved a better performance than the other state-of-the-art ones.

However, domain knowledge may be represented by there are different types of the constraints in applications. This paper only deal with domain knowledge that can be represented by hard constraints such as road edges. Some types of domain knowledge cannot be expressed in this way such as speed limit. Instead of representing as a hard constraint, it sets a probability likelihood (in the range of [0,1]) for the state vector being in different regions, e.g. soft constraint [14] Extending the current algorithm to incorporate with soft constraints is the next step of the research.

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