Calibration of Multi-Target Tracking Algorithms Using Non-Cooperative Targets

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***Abstract*—Tracking systems are based on models, in particular, the target dynamics model and the sensor measurement model. In most practical situations the two models are not known exactly and are typically parametrized by an unknown random vector θ. The paper proposes a Bayesian algorithm based on importance sampling for the estimation of the static parameter θ. The input are measurements collected by the tracking system, with non-cooperative targets present in the surveillance volume during the data acquisition. The algorithm relies on the particle filter implementation of the probability density hypothesis (PHD) filter to evaluate the likelihood of θ. Thus, the calibration algorithm, as a byproduct, also provides a multi-target state estimate. An application of the proposed algorithm to translational sensor bias estimation is presented in detail as an illustration. The resulting sensor-bias estimation method is applicable to asynchronous sensors and does not require prior knowledge of measurement-to-target associations.**

***Index Terms*—Bayesian estimation, calibration, importance sampling, PHD filter, sensor bias estimation, target tracking.**

I. Introduction

MULTI-TARGET multi-sensor tracking systems [1] are based on mathematical models which typically include many parameters. The two main models are the target dynamic model (for target birth and motion) and the sensor measurement model. The typical parameters used in the two models are the process noise level, the false alarm rate, the probability of detection, sensor biases, various factors such as the propagation losses, receiver gains, etc. Calibration of tracking systems, through estimation of their model parameters, is an important prerequisite for their operational deployment. Yet, apart from sensor registration and clutter estimation, calibration has received little attention by the research community and is mainly done in an ad-hoc manner.

This paper is devoted to the estimation of static parameters which may feature in the probabilistic models that describe target dynamics (the transitional density) and sensor measurements (the likelihood function). Let a random vector $\boldsymbol{\theta} \in \Theta$ represent the (static) vector of parameters of interest for estimation/calibration. The paper proposes a batch method for Bayesian estimation of the parameter vector $\boldsymbol{\theta}$. Since $\boldsymbol{\theta}$ is unknown we ascribe to it a prior density and carry out Bayesian inference on the joint space of the multi-target dynamic state and the parameter vector $\boldsymbol{\theta}$. The multi-target state, modeled by a random finite set [2], determines the number of targets and their location in the target state space at a particular time. The multi-target state, conditioned on $\boldsymbol{\theta}$, is estimated using the first moment approximation of the Bayesian multi-target filter, known as the probability hypothesis density (PHD) filter [3]. The data used for estimation of $\boldsymbol{\theta}$ are the sets of measurements collected by the sensor(s) on non-cooperative targets, which happen to be in the surveillance volume during the data collection interval.

The approach is motivated by a new class of PHD filters based on hierarchical point process models [4]. The resulting framework is general and hence applicable to a variety of problems characterized by a hierarchy of two point processes, the parent and its offsprings. So far it has been applied to solve the following problems: tracking groups of targets [5], tracking an extended target [6], simultaneous localization and mapping (SLAM) [7], [8]. The preliminary results of a recursive approach to calibration of tracking algorithms were reported in [9]. The main advantage of the family of algorithms based on hierarchical models is that it treats the two point processes separately but interactively, thereby avoiding computationally expensive joint estimation on an augmented state space.

Calibration of tracking algorithms using the PHD filter has been considered recently in the context of sensor bias estimation [10], [11] and clutter estimation [12], [13]. In order to solve the sensor bias estimation, Lian *et al.* [10] and Mahler and El-Fallah [11] proposed to augment the target state space of the PHD filter with the bias vector. Our approach is distinct from these approaches since we approximate the prior with a single-cluster Poisson process. The mathematical expression for the likelihood is similar to that used in the maximum likelihood clutter map estimation algorithm proposed in [12]. Our method differs from [12] in the fact that we use this likelihood for estimation of the bias parameter within a Bayesian paradigm. Augmented target state spaces have also been used in [13] for the purpose of estimation of the probability of detection.

The proposed Bayesian estimator of tracking parameters cannot be derived in a closed-form and is therefore approximately solved by a two-layer Monte-Carlo method. Parameter estimation, in the upper level, is carried out using the importance sampling method [14]. The key component at this level

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plays the likelihood function of the measurement data set. This likelihood is provided by the PHD filter, which is implemented in the sequential Monte Carlo framework, as a particle filter.

Monte Carlo batch methods for parameter estimation of dynamic stochastic systems have been studied in the past [15]; see also [16, Sec.IV] and references therein, for an overview. One such a method, known as the particle MCMC [17], [18] has particularly become popular among the practitioners. These methods, however, are formulated and solved in the context of a single-target in the absence of false detections. Our paper can therefore be seen as an extension from single-target to multi-target nonlinear non-Gaussian Monte Carlo parameter estimation, in the presence of clutter. This extension is difficult because it has to cope with additional uncertainty due to false detections, missed-detections and an unknown number of appearing/disappearing targets.

After describing the algorithm in the general framework of parameter estimation of a tracking system, the paper focuses on a particular problem: estimation of translational biases in measurements from multiple sensors. Whereas most standard algorithms for multi-sensor bias estimation require access to track-associated observations from the sensors [19], [20], (some even require the multi-sensor observations to be synchronous [21]), the proposed sensor registration method is free of such restrictions.

The paper is organized as follows. The background to the problem and its formulation are given in Section II. The conceptual solution and its Monte Carlo implementation are presented in Section III. The application of the proposed theoretical solution to the problem of multi-sensor bias estimation, with a numerical example, is described in Section IV. Finally the conclusions are drawn in Section V.

II. Background and Problem Formulation

Suppose at time $k \in \mathbb{N}$ there are n_k targets with states $\mathbf{x}_{k,1}, \ldots, \mathbf{x}_{k,n_k}$ taking values in the state space $\mathcal{X} \subseteq \mathbb{R}^{n_x}$. Both the number of targets n_k and their individual states in $\mathcal{X}$ are random and time-varying. The multi-target state, represented by a finite set

$$\mathbf{X}_k = \{\mathbf{x}_{k,1}, \ldots, \mathbf{x}_{k,n_k}\} \in \mathcal{F}(\mathcal{X}), \tag{1}$$

can conveniently be modeled as a random finite set on $\mathcal{X}$. Here $\mathcal{F}(\mathcal{X})$ is a set of finite subsets of $\mathcal{X}$. The targets are non-cooperative and their states are unknown.

The detection process is imperfect and some of the targets in $\mathbf{X}_k$ are detected, while the others are missed. In addition, the detector typically creates some false detections. Suppose at time k there are m_k detections in the measurement set, each taking a value in the observation space $\mathcal{Z} \subseteq \mathbb{R}^{n_z}$. Then the multi-target observation set,

$$\mathbf{Z}_k = \{\mathbf{z}_{k,1}, \ldots, \mathbf{z}_{k,m_k}\} \in \mathcal{F}(\mathcal{Z}), \tag{2}$$

whose cardinality and individual points in the measurement space are random, is also modeled by a random finite set. $\mathcal{F}(\mathcal{Z})$ is a set of finite subsets of $\mathcal{Z}$.

The hidden multi-target Markov state process is characterized by its initial density $\mathbf{X}_1 \sim \mu(\mathbf{X}|\boldsymbol{\theta})$ and the multi-target transitional density $\Pi(\mathbf{X}_k|\mathbf{X}_{k-1}, \boldsymbol{\theta})$, for some static parameter $\theta \in \Theta$. The multi-object process is not observed directly, but through the observation process. The observation process is assumed to be conditionally independent given the multi-target state process, and fully specified by the multi-target likelihood $\varphi(\mathbf{Z}_k|\mathbf{X}_k, \boldsymbol{\theta})$, again conditioned on $\theta \in \Theta$. Standard cases of multi-target densities, $\Pi(\mathbf{X}_k|\mathbf{X}_{k-1}, \boldsymbol{\theta})$ and $\varphi(\mathbf{Z}_k|\mathbf{X}_k, \boldsymbol{\theta})$, are derived in [2]. The unknown parameters of $\Pi(\mathbf{X}_k|\mathbf{X}_{k-1}, \boldsymbol{\theta})$ may include the process noise variance, birth and survival probabilities, birth process intensity, etc. The unknown parameters of $\varphi(\mathbf{Z}_k|\mathbf{X}_k, \boldsymbol{\theta})$ may include sensor or environmental characteristics, such as the probability of detection, propagation factors, clutter parameters, sensor biases, measurement noise variances, etc.

The problem is to estimate the posterior density $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$, where $\mathbf{Z}_{1:K} \equiv \mathbf{Z}_1, \ldots, \mathbf{Z}_K$ is the observation set sequence accumulated over time steps $k = 1, \ldots, K$. Having the prior $p_0(\boldsymbol{\theta})$, the solution in the Bayesian framework is $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K}) \propto f(\mathbf{Z}_{1:K}|\theta)p_0(\boldsymbol{\theta})$. The computation of $f(\mathbf{Z}_{1:K}|\boldsymbol{\theta})$, however, will require us to perform the inference on the joint space of the parameter vector and the multi-target trajectory (history) $\mathbf{X}_{1:K}$. Thus we can consider our problem as a component of a broader problem where the goal is to find the posterior density:

$$f\left(\boldsymbol{\theta}, \mathbf{X}_{1:K}|\mathbf{Z}_{1:K}\right) \propto f\left(\mathbf{X}_{1:K}|\mathbf{Z}_{1:K}, \boldsymbol{\theta}\right) f\left(\mathbf{Z}_{1:K}|\boldsymbol{\theta}\right) p_0(\boldsymbol{\theta}) \tag{3}$$

where $f(\mathbf{X}_{1:K}|\mathbf{Z}_{1:K}, \boldsymbol{\theta})$ is the posterior of the multi-target trajectory given all observation sets and conditioned on $\boldsymbol{\theta}$.

III. The Proposed Solution

The proposed solution follows the same line of thought as the Monte Carlo batch techniques for parameter estimation of stochastic dynamic systems, reviewed in [16, Sec.IV]. The key idea is to run a sequential Monte Carlo method to obtain an estimate of $f(\mathbf{X}_{1:K}|\mathbf{Z}_{1:K}, \boldsymbol{\theta})$ for a given value of $\boldsymbol{\theta}$. This provides a simple way of evaluating the observation likelihood $f(\mathbf{Z}_{1:K}|\boldsymbol{\theta})$ and thus to a practical solution for $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$.

In practice, however, the difficulty is that the sequential Monte Carlo implementation of the multi-target Bayes filter, which estimates $f(\mathbf{X}_{1:K}|\mathbf{Z}_{1:K}, \boldsymbol{\theta})$, can be implemented only for a small number of targets [22], [23], [24]. This is because its state includes all individual states of the existing targets at the time, and consequently its computational complexity grows exponentially with the number of targets. For this reason we propose to approximate (3) with its first-order moment (expected value) with respect to $\mathbf{X}_{1:K}$, also known as the intensity function or the probability hypothesis density (PHD).

Using the relationship between the PHD and the multi-target density [2, Eq.(.16.14)], we replace (3) with (see Appendix for further explanation):

$$D\left(\boldsymbol{\theta}, \mathbf{x}_{1:K}|\mathbf{Z}_{1:K}\right) \propto D\left(\mathbf{x}_{1:K}|\mathbf{Z}_{1:K}, \boldsymbol{\theta}\right) f\left(\mathbf{Z}_{1:K}|\boldsymbol{\theta}\right) p_0(\boldsymbol{\theta}) \tag{4}$$

where D denotes the PHD. Note that the posterior PHD $D(\mathbf{x}_{1:K}|\mathbf{Z}_{1:K}, \boldsymbol{\theta})$, as opposed to the posterior density $f(\mathbf{X}_{1:K}|\mathbf{Z}_{1:K}, \boldsymbol{\theta})$, is defined on a single-target state space over time. The sequential Monte Carlo implementation of the PHD filter (also known as the *particle PHD filter*), which recursively estimates the posterior PHD $D(\mathbf{x}_k|\mathbf{Z}_{1:k}, \boldsymbol{\theta})$, has received widespread attention, eg. [25]–[28]. It will play the crucial role in estimation of $\boldsymbol{\theta}$.

A. PHD Filter

For completeness this section reviews the PHD filter for the recursive estimation of $D(\mathbf{x}_k|\mathbf{Z}_{1:k},\boldsymbol{\theta})$. The prediction equation of the PHD filter is given by [3]:

$$D(\mathbf{x}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta}) = \gamma(\mathbf{x}_k|\boldsymbol{\theta}) + \int p_{\mathrm{S}}(\mathbf{x}_{k-1}|\boldsymbol{\theta})\,\tau(\mathbf{x}_k|\mathbf{x}_{k-1},\boldsymbol{\theta}) \times D(\mathbf{x}_{k-1}|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})\,d\mathbf{x}_{k-1} \quad (5)$$

where: $\gamma(\mathbf{x}_k|\boldsymbol{\theta})$ is the PHD of new target births between time $k-1$ and k; $p_{\mathrm{s}}(\mathbf{x}_{k-1}|\boldsymbol{\theta})$ is the probability that an object in state $\mathbf{x}_{k-1}$ will survive until time k; $\tau(\mathbf{x}_k|\mathbf{x}_{k-1},\boldsymbol{\theta})$ is the single-target transitional density. The update equation of the PHD filter, applied upon receiving the measurement set $\mathbf{Z}_k$, is given by[3]:

$$D(\mathbf{x}_k|\mathbf{Z}_{1:k},\boldsymbol{\theta}) = (1-p_{\mathrm{D}}(\mathbf{x}_k;\boldsymbol{\theta}))\,D(\mathbf{x}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta}) + \sum_{\mathbf{z}_k\in\mathbf{Z}_k}\frac{p_{\mathrm{D}}(\mathbf{x}_k|\boldsymbol{\theta})\,g(\mathbf{z}_k|\mathbf{x}_k,\boldsymbol{\theta})D(\mathbf{x}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})}{\kappa(\mathbf{z}_k|\boldsymbol{\theta})+\int p_{\mathrm{D}}(\mathbf{x}_k|\boldsymbol{\theta})\,g(\mathbf{z}_k|\mathbf{x}_k,\boldsymbol{\theta})D(\mathbf{x}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})\,d\mathbf{x}_k} \quad (6)$$

where: $p_{\mathrm{D}}(\mathbf{x}|\boldsymbol{\theta})$ is the probability that a target in state $\mathbf{x}_k$ will be detected at time k; $g(\mathbf{z}_k|\mathbf{x}_k,\boldsymbol{\theta})$ is the single-target measurement likelihood and $\kappa(\mathbf{z}_k|\boldsymbol{\theta})$ is the PHD of clutter. The integral of the PHD $D(\mathbf{x}_k|\mathbf{Z}_{1:k},\boldsymbol{\theta})$ over $\mathcal{X}$ is the posterior expectation of the number of targets in the set $\mathbf{X}_k$.

The observation likelihood $f(\mathbf{Z}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})$, defined as [3]:

$$f(\mathbf{Z}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta}) = \int f(\mathbf{Z}_k|\mathbf{X}_k,\boldsymbol{\theta})\,f(\mathbf{X}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})\,\delta\mathbf{X}_k, \quad (7)$$

(note that the integral here is a set-integral [2]), is a by-product of the PHD filter. It can be expressed as [3, Eq.(116)]:

$$f(\mathbf{Z}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta}) = \alpha\exp\left\{-\int p_{\mathrm{D}}(\mathbf{x}_k;\boldsymbol{\theta})\,(\mathbf{x}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})\,d\mathbf{x}_k\right\} \times \prod_{\mathbf{z}_k\in\mathbf{Z}_k}\left(\kappa(\mathbf{z}_k|\boldsymbol{\theta}) + \int p_{\mathrm{D}}(\mathbf{x}_k|\boldsymbol{\theta})\,g(\mathbf{z}_k|\mathbf{x}_k,\boldsymbol{\theta})\,D(\mathbf{x}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})\,d\mathbf{x}_k\right) \quad (8)$$

where α is a constant of proportionality, which will be shown later to be unimportant (it cancels out). Therefore, without loss of generality, we adopt $\alpha = 1$. One can compute $f(\mathbf{Z}_{1:K}|\boldsymbol{\theta})$, the likelihood of the measurement set sequence which features in (4), using (8) and the decomposition:

$$f(\mathbf{Z}_{1:K}|\boldsymbol{\theta}) = \prod_{k=1}^{K} f(\mathbf{Z}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta}). \quad (9)$$

It can be easily verified that the conventional, single-target recursive Bayesian estimator (nonlinear filter) [29] is a special case of the PHD filter. In this special case, the target exists (i.e. a dynamic system is turned on) all the time and hence $p_{\mathrm{S}} = 1$ and $\gamma = 0$. Furthermore, detection is perfect and therefore $p_{\mathrm{D}} = 1$ and $\kappa = 0$. Consequently both $\mathbf{X}_k$ and $\mathbf{Z}_k$ are singletons, and can be replaced by $\mathbf{x}_k$ and $\mathbf{z}_k$, respectively. Finally, the integral of the posterior PHD over the state space equals one, and therefore the posterior PHD is identical to the posterior density function.

B. Particle PHD Filter

The particle PHD filter approximates the PHD $D(\mathbf{x}_k|\mathbf{Z}_{1:k},\boldsymbol{\theta})$ by a weighted set of samples (particles) $\{\mathbf{x}_k^{(i)}, w_k^{(i)}\}_{1\le i\le N_k}$, where the weights satisfy $w_k^{(i)} \ge 0$, $i = 1,\ldots,N$. The sum of the weights results in an estimate of the expected number of targets. Next we briefly summarize the particle PHD filter implementation following [25].

Let the particle approximation of $D(\mathbf{x}_{k-1}|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})$ be:

$$\widehat{D}(\mathbf{x}_{k-1}|\mathbf{Z}_{1:k-1},\boldsymbol{\theta}) = \sum_{i=1}^{N_{k-1}} w_{k-1}^{(i)}\delta_{\mathbf{x}_{k-1}^{(i)}}(\mathbf{x}_{k-1}) \quad (10)$$

Then the particle approximation of the predicted PHD $D(\mathbf{x}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta})$ is:

$$\widehat{D}(\mathbf{x}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta}) = \sum_{i=1}^{N_{k-1}+L_k} w_{k|k-1}^{(i)}\delta_{\mathbf{x}_{k|k-1}^{(i)}}(\mathbf{x}_k) \quad (11)$$

where

$$\mathbf{x}_{k|k-1}^{(i)} \sim \begin{cases} q(\mathbf{x}_k|\mathbf{x}_{k-1},\mathbf{Z}_k,\boldsymbol{\theta}), & i = 1,\ldots,N_{k-1} \\ p(\mathbf{x}_k|\mathbf{Z}_k,\boldsymbol{\theta}) & i = N_{k-1}+1,\ldots,N_{k-1}+L_k \end{cases} \quad (12)$$

$$w_{k|k-1}^{(i)} = \begin{cases} \dfrac{p_{\mathrm{S}}\left(\mathbf{x}_{k|k-1}^{(i)}|\boldsymbol{\theta}\right)\tau\left(\mathbf{x}_{k|k-1}^{(i)}|\mathbf{x}_{k-1}^{(i)},\boldsymbol{\theta}\right)w_{k-1}^{(i)}}{q\left(\mathbf{x}_{k|k-1}^{(i)}|\mathbf{x}_{k-1}^{(i)},\mathbf{Z}_k,\boldsymbol{\theta}\right)}, & i=1,\ldots,N_{k-1} \\ \dfrac{\gamma\left(\mathbf{x}_{k|k-1}^{(i)}|\boldsymbol{\theta}\right)}{L_k\,p\left(\mathbf{x}_{k|k-1}^{(i)}|\mathbf{Z}_k,\boldsymbol{\theta}\right)}, & i=N_{k-1}+1,\ldots,N_{k-1}+L_k \end{cases} \quad (13)$$

Here $q(\mathbf{x}_k|\mathbf{x}_{k-1},\mathbf{Z}_k,\boldsymbol{\theta})$ and $p(\mathbf{x}_k|\mathbf{Z}_k,\boldsymbol{\theta})$ are the importance (proposal) densities for the surviving and newborn targets, respectively, while L_k is the number of newborn targets particles. The bootstrap PHD particle filter (which we assume onwards) replaces the proposal $q(\mathbf{x}_k|\mathbf{x}_{k-1},\mathbf{Z}_k,\boldsymbol{\theta})$ with the single-target transitional density $\tau(\mathbf{x}_k|\mathbf{x}_{k-1},\boldsymbol{\theta})$. The design of $p(\mathbf{x}_k|\mathbf{Z}_k,\boldsymbol{\theta})$ is discussed in [27]. The birth intensity $\gamma(\mathbf{x}_k|\boldsymbol{\theta})$ is then typically replaced by $n_b \times p(\mathbf{x}_k|\mathbf{Z}_k,\boldsymbol{\theta})$, where $n_b \in \mathbb{R}^+$ is the expected number of target births in interval from $k-1$ to k. Consequently, the birth particle weights are uniform.

According to (6), the particle weights are updated as follows: (see equation (14) at the bottom of next page). At this stage it is also convenient to estimate the expression in (8), that is

$$\widehat{f}(\mathbf{Z}_k|\mathbf{Z}_{1:k-1},\boldsymbol{\theta}) = \exp\left\{-\sum_{j=1}^{N_{k-1}+L_k} p_{\mathrm{D}}\left(\mathbf{x}_{k|k-1}^{(j)};\boldsymbol{\theta}\right)w_{k|k-1}^{(j)}\right\} \times \prod_{\mathbf{z}_k\in\mathbf{Z}_k}\left(\kappa(\mathbf{z}_k|\boldsymbol{\theta}) + \sum_{j=1}^{N_{k-1}+L_k} p_{\mathrm{D}}\left(\mathbf{x}_{k|k-1}^{(j)}|\boldsymbol{\theta}\right) \times g\left(\mathbf{z}_k|\mathbf{x}_{k|k-1}^{(j)},\boldsymbol{\theta}\right)w_{k|k-1}^{(j)}\right) \quad (15)$$

The estimate of the number of targets in $\mathbf{X}_k$ is computed as

$$\widehat{n}_k = \sum_{i=1}^{N_{k-1}+L_k} \tilde{w}_k^{(i)}.$$

Finally, the particles are resampled N_k times from $\{(\tilde{w}_k^{(i)}/\widehat{n}_k), \mathbf{x}_{k|k-1}^{(i)}\}_{1\leq i\leq N_{k-1}+L_k}$ to obtain a new particle set $\{w_k^{(i)} = (\widehat{n}_k/N_k), \mathbf{x}_k^{(i)}\}_{1\leq i\leq N_k}$ as an approximation of $D(\mathbf{x}_k|\mathbf{Z}_{1:k},\boldsymbol{\theta})$, the PHD at time k. Following the resampling step, the MCMC move step [30, p.55] is usually necessary to increase the diversity of particles.

Note that (4), as opposed to (3), does not provide the posterior over the multi-target history $\mathbf{X}_{1:K}$, but over $\mathbf{x}_{1:K}$. Although we are primarily interested in the estimation of $\boldsymbol{\theta}$, an estimate of the multi-target history $\widehat{\mathbf{X}}_{1:K}$ may be desirable as a by-product. The particle PHD filter can provide this estimate. Details of an accurate and fast method for the estimation of multi-target state from a particle representation of the PHD (10) are given in [31, Sec.4].

C. Importance Sampling With Progressive Correction

This subsection deals with the computation of the posterior distribution $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$, whose solution in the Bayesian framework is given by:

$$p(\boldsymbol{\theta}|\mathbf{Z}_{1:K}) \propto f(\mathbf{Z}_{1:K}|\boldsymbol{\theta})\, p_0(\boldsymbol{\theta}) \tag{16}$$

An estimate of the likelihood function $f(\mathbf{Z}_{1:K}|\boldsymbol{\theta})$ is provided by the particle PHD filter, as described in (8), (9) and (15). The solution to (16), however, cannot be found in the closed form and again is approximated by a Monte Carlo technique. This time we apply an importance sampling method with progressive correction.

Quantities of interest related to $\boldsymbol{\theta}$ can be computed from the posterior, for example, the posterior mean is

$$\mathrm{E}(\boldsymbol{\theta}|\mathbf{Z}_{1:K}) = \int \boldsymbol{\theta} p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})\, d\boldsymbol{\theta}. \tag{17}$$

Approximation of (17) via importance sampling involves drawing a sample $\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_M$, where M is the sample size, from an importance density q and approximating the integral in (17) by a sum,

$$\widehat{\boldsymbol{\theta}} = \sum_{j=1}^{M} \ell_j \boldsymbol{\theta}_j. \tag{18}$$

The weights $\ell_j \geq 0$, $j=1,\ldots,M$ are specified as:

$$\ell_j = \frac{\tilde{\ell}_j}{\sum_{m=1}^{M}\tilde{\ell}_m}, \text{ with } \tilde{\ell}_j = \frac{f(\mathbf{Z}_{1:K}|\boldsymbol{\theta}_j)\, p_0(\boldsymbol{\theta}_j)}{q(\boldsymbol{\theta}_j)} \tag{19}$$

Note that the calculation of weights requires only a function which is proportional to the posterior density. As discussed earlier, we can only compute an estimate of $f(\mathbf{Z}_{1:K}|\boldsymbol{\theta}_j)/\alpha^K$ using the particle PHD filter. Now we see why the factor of proportionality α is not important: it cancels out by the normalization of weights.

For a large class of importance densities q, approximation (18) becomes increasingly accurate as the sample size $M \to \infty$. However, for a finite M, the accuracy of the approximation depends greatly on the particular importance density. A desirable property of an importance density is that it produces weights with a small variance. Equation (19) implies that a good importance density should resemble the posterior. Since the posterior is unknown and importance sampling using the prior is inaccurate for any reasonable sample size, we propose to apply importance sampling with the progressive correction [32].

The progressive correction constructs a sequence of target distributions from which we draw samples sequentially. The first target distribution is typically similar to the prior, while the final target distribution is the posterior. The consecutive target distributions in the sequence should not differ too greatly. Similar ideas have been proposed under different terms, e.g. tempering [14, p.540] and bridging densities [33]. The particle flow with log-homotopy [34] achieves the same result, without the need for resampling.

Let S denote the number of stages and π_s, $s=1,\ldots,S$, denote the target distribution for the sth stage. Note that at the final stage S we have $\pi_S = p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$. A series of target distributions which starts from the prior and gradually becomes similar to the posterior can be constructed by setting, for $s=1,\ldots,S$

$$\pi_s(\boldsymbol{\theta}) \propto [f(\mathbf{Z}_{1:K}|\boldsymbol{\theta})]^{\Lambda_s}\, p_0(\boldsymbol{\theta}) \tag{20}$$

where $\Lambda_s = \sum_{j=1}^{s}\lambda_j$ with $\lambda_j \in (0,1]$ and $\Lambda_S = 1$. In this way Λ_s is an increasing function of s, upper bounded by one. Note that the intermediate likelihood used for $s < S$ will be broader (more spread) than the true likelihood, particularly in the early stages. In the later stages the intermediate likelihood sharpens (becomes more focused) so that the sample gradually concentrates in the area of the parameter space suggested by the true likelihood.

The procedure for sequentially drawing samples from target distributions $\pi_1,\pi_2,\ldots,\pi_S$ is described next. Suppose a sample $\{\boldsymbol{\theta}_j^{s-1}\}_{1\leq j\leq M}$ from π_{s-1} is available and we wish to produce a sample from π_s. Note that for $s=1$, the sample $\{\boldsymbol{\theta}_j^{s-1}\}_{1\leq j\leq M}$ is drawn from the prior $p_0=\pi_0$. The first step is to compute weights for samples in $\{\boldsymbol{\theta}_j^{s-1}\}_{1\leq j\leq M}$ as: $\tilde{\ell}_j^s = f(\mathbf{Z}_{1:K}|\theta_j^{s-1})^{\lambda_s}$ for $j=1,\ldots,M$. The weights are then normalized, i.e. for $j=1,\ldots,M$,

$$\ell_j^s = \frac{\tilde{\ell}_j^s}{\sum_{m=1}^{M}\tilde{\ell}_m^s}$$

$$\tilde{w}_k^{(i)} = \left(1 - p_{\mathrm{D}}\left(\mathbf{x}_{k|k-1}^{(i)};\boldsymbol{\theta}\right)\right) w_{k|k-1}^{(i)} + \sum_{\mathbf{z}_k\in\mathbf{Z}_k} \frac{p_{\mathrm{D}}\left(\mathbf{x}_{k|k-1}^{(i)}|\boldsymbol{\theta}\right) g\left(\mathbf{z}_k|\mathbf{x}_{k|k-1}^{(i)},\boldsymbol{\theta}\right) w_{k|k-1}^{(i)}}{\kappa(\mathbf{z}_k|\boldsymbol{\theta}) + \sum_{j=1}^{N_{k-1}+L_k} p_{\mathrm{D}}\left(\mathbf{x}_{k|k-1}^{(j)}|\boldsymbol{\theta}\right) g\left(\mathbf{z}_k|\mathbf{x}_{k|k-1}^{(j)},\boldsymbol{\theta}\right) w_{k|k-1}^{(j)}} \tag{14}$$

In order to derive any benefits from tempering, it is necessary to remove the lower weighted members of the sample $\{\boldsymbol{\theta}_j^{s-1}\}_{1\leq j\leq M}$ and diversify the remaining ones. Hence the next step is resampling, followed by the MCMC move step.

Resampling involves selecting M indices $i_1^s, \ldots, i_M^s$ such that $Pr\{i_j^s = j\} = \ell_j^s$. The resampled sample $\{\tilde{\boldsymbol{\theta}}_j^s\}_{1\leq j\leq M}$ will almost certainly contain duplicate members since the weights $\ell_1^s, \ldots, \ell_M^s$ will most likely be uneven. In order to remove duplication and thus increase the sample diversity, the MCMC move step is applied to the resampled sample $\{\tilde{\boldsymbol{\theta}}_j^s\}_{1\leq j\leq M}$. For each member of $\{\tilde{\boldsymbol{\theta}}_j^s\}_{1\leq j\leq M}$, a new sample member is proposed as a draw

$$\boldsymbol{\theta}_j^{s,*} \sim g_s\left(\cdot|\tilde{\boldsymbol{\theta}}_j^s\right)$$

where g_s denotes the proposal density for the sth stage. Then the Metropolis-Hastings scheme is applied, whereby $\boldsymbol{\theta}_j^{s,*}$ is accepted with certain probability or rejected. In this way we form a new sample $\{\boldsymbol{\theta}_j^s\}_{1\leq j\leq M}$, where $\boldsymbol{\theta}_j^s = \boldsymbol{\theta}_j^{s,*}$, if the move is accepted, and $\boldsymbol{\theta}_j^s = \tilde{\boldsymbol{\theta}}_j^s$, if the move is rejected. The acceptance probability is adopted as

$$\eta = \min\left\{1, \frac{\pi_s\left(\boldsymbol{\theta}_j^{s,*}\right) g_s\left(\tilde{\boldsymbol{\theta}}_j^s|\boldsymbol{\theta}_j^{s,*}\right)}{\pi_s\left(\tilde{\boldsymbol{\theta}}_j^s\right) g_s\left(\boldsymbol{\theta}_j^{s,*}|\tilde{\boldsymbol{\theta}}_j^s\right)}\right\}.$$

The proposal density g_s should produce candidates over a reasonably large area of the parameter space (in order to increase diversity), but within the area of high likelihood. A suitable proposal can be selected as follows [35]:

$$g_s(\boldsymbol{\theta}^*|\boldsymbol{\theta}) = g_s(\boldsymbol{\theta}^*) = \mathcal{N}(\boldsymbol{\theta}^*; \hat{\boldsymbol{\mu}}_s, \varepsilon\hat{\mathbf{C}}_s) \tag{21}$$

where $\mathcal{N}(\boldsymbol{\theta}; \boldsymbol{\mu}, \mathbf{C}) = \exp[-(\boldsymbol{\theta}-\boldsymbol{\mu})^\intercal \mathbf{C}^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu})/2]/\sqrt{|2\pi\mathbf{C}}$ is the Gaussian distribution; $\hat{\boldsymbol{\mu}}_s$ and $\hat{\mathbf{C}}_s$ are the sample mean and covariance matrix, respectively, of a weighted sample $\{\ell_j^s, \boldsymbol{\theta}_j^{s-1}\}_{1\leq j\leq M}$, and ε is a user defined parameter.

The computational expense of tempering depends on the number of stages S and the correction factors $\lambda_1, \ldots, \lambda_S$. While a small value of S is favored for computational reasons, the successive intermediate distributions π_s are made more similar by choosing large S. An adaptive scheme proposed in [32] is used here to balance the conflicting requirements for S.

The pseudo-code of the proposed algorithm for joint estimation of $\boldsymbol{\theta}$ and $\mathbf{X}_{1:K}$ is given in Alg.1. It incorporates the adaptive selection of correction factors λ_s, in lines 11 and 12. Here $1 \ll H \leq M$ and $\phi > 0$ are user defined parameters which control the number and values of correction factors. The reasoning behind the adaptive scheme for the selection of correction factors is explained in [32]: the increments are smaller if particles are characterized by small values of likelihoods, and vice versa. A smaller value of ϕ or a larger value of H, reduces the increment between the correction factors and thus increases the number of stages S.

The proposed parameter estimation algorithm is primarily designed to compute the sample $\{\boldsymbol{\theta}_j^S\}_{1\leq j\leq M}$ which approximates the posterior $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$. As a by-product, however, it can also compute an estimate of the multi-target history $\widehat{\mathbf{X}}_{1:K}$ (see lines 30 and 31).

IV. Application to Sensor Bias Estimation

Multi-sensor bias estimation has received a considerable interest by the tracking community, see for example [19]–[21] and references therein. This section illustrates the proposed algorithm as a solution to multi-sensor translational bias estimation using targets of opportunity. The sensors can operate asynchronously with imperfect detection ($p_{\mathrm{D}} < 1$ and false alarms), the targets can dynamically appear and disappear from the surveillance volume, and the usual requirement in sensor bias estimation, to know the association of measurements to targets, is not required.

Algorithm 1 Parameter estimation using particle PHD importance sampling

1: **Input**: Accumulated measurement sets $\mathbf{Z}_{1:K}$; prior p_0
2: **Initialization**:
3: Set $s \leftarrow 0$, $\Lambda_0 \leftarrow 0$
4: **for** $j = 1, \ldots, M$ **do**
5: Draw $\boldsymbol{\theta}_j^0 \sim p_0$
6: Run particle the PHD filter using $\boldsymbol{\theta}_j^0$ to estimate log-likelihood $\psi_j^0 = \log f(\mathbf{Z}_{1:K}|\boldsymbol{\theta}_j^0)$
7: **end for**
8: **Progressive correction**:
9: **while** $\Lambda_s < 1$ and $s < S$ **do**
10: $s \leftarrow s+1$
11: Sort negative log-likelihoods: $-\psi_{(1)}^{s-1} < -\psi_{(2)}^{s-1} < \cdots - \psi_{(M)}^{s-1}$
12: $\lambda_s \leftarrow \min\{1-\Lambda_{s-1}, -\phi/\psi_{(H)}^{s-1}\}$
13: $\Lambda_s \leftarrow \Lambda_{s-1} + \lambda_s$
14: For $j = 1, \ldots, M$, compute $\tilde{\ell}_j^s = \exp\{\lambda_s \cdot \psi_j^{s-1}\}$
15: Normalization: $\ell_j^s = \tilde{\ell}_j^s / \sum_{m=1}^M \tilde{\ell}_m^s$
16: Compute the sample mean $\hat{\boldsymbol{\mu}}_s$ and covariance matrix $\hat{\mathbf{C}}_s$
17: **for** $j = 1, \ldots, M$ **do**
18: Select $i_j = j$ with probability ℓ_j^s and set $\tilde{\boldsymbol{\theta}}_j^s = \boldsymbol{\theta}_{i_j}^{s-1}$ and $\tilde{\psi}_j^s = \psi_{i_j}^{s-1}$
19: Draw $\boldsymbol{\theta}_j^{s,*} \sim g_s(\cdot; \hat{\boldsymbol{\mu}}_s, \hat{\mathbf{C}}_s)$
20: Run the particle PHD filter using $\boldsymbol{\theta}_j^{s,*}$ to estimate $\psi_j^{s,*} = \log f(\mathbf{Z}_{1:K}|\boldsymbol{\theta}_j^{s,*})$
21: Compute acceptance probability $\eta = \min\{1, \exp[\lambda_s(\psi_j^{s,*} - \tilde{\psi}_j^s)] g_s(\tilde{\boldsymbol{\theta}}_j^s)/g_s(\boldsymbol{\theta}_j^{s,*})\}$
22: Draw $u \sim \mathcal{U}_{[0,1]}$
23: **if** $u < \eta$ **then**
24: Set $\boldsymbol{\theta}_j^s = \boldsymbol{\theta}_j^{s,*}$ and $\psi_j^s = \psi_j^{s,*}$
25: **else**
26: Set $\boldsymbol{\theta}_j^s = \tilde{\boldsymbol{\theta}}_j^s$ and $\psi_j^s = \tilde{\psi}_j^s$
27: **end if**
28: **end for**
29: **end while**
30: Compute the mean or the maximum a posteriori estimate $\widehat{\boldsymbol{\theta}}$ from the sample $\{\boldsymbol{\theta}_j^S\}_{1\leq j\leq M}$
31: Run particle PHD filter using $\widehat{\boldsymbol{\theta}}$ to obtain an estimate of the multi-target history $\widehat{\mathbf{X}}_{1:K}$
32: **Output**: Sample $\{\boldsymbol{\theta}_j^S\}_{1\leq j\leq M}$ and $\widehat{\mathbf{X}}_{1:K}$

A. Specification of the Bias Estimation Problem

Let us adopt a 2D scenario, where measurements are collected by two static sensors with overlapping coverage. The state vector of each individual target is $\mathbf{x} = [x\ \dot{x}\ y\ \dot{y}]^\top$, where (x, y) denotes the position of the target and $(\dot{x}, \dot{y})$ its velocity. Sensor measurements are affected by two sources of error: the systematic error (or bias) and stochastic error (measurement noise). Let the measurement set reported at time k by sensor $r_k \in \{1, 2\}$ be denoted $\mathbf{Z}_k^{(r_k)}$. For a measurement $\mathbf{z} \in \mathbf{Z}_k^{(r_k)}$ which originates from an object $\mathbf{x} \in \mathbf{X}_k$, the single-object likelihood is given by:

$$g_k^{(r_k)}(\mathbf{z}|\mathbf{x}, \boldsymbol{\theta}) = \mathcal{N}\left(\mathbf{z}; h_k^{(r_k)}(\mathbf{x}) - \boldsymbol{\theta}^{(r_k)}, \boldsymbol{\Sigma}_k^{(r_k)}\right). \tag{22}$$

We further assume that sensors provide range and azimuth measurements and hence:

$$h_k^{(r)}(\mathbf{x}) = \begin{bmatrix} \sqrt{(x - x_r)^2 + (y - y_r)^2} \\ \arctan\left(\frac{x - x_r}{y - y_r}\right) \end{bmatrix}, \tag{23}$$

where (x_r, y_r) is the position of sensor r. Vector $\boldsymbol{\theta}$ consist of four components, i.e.

$$\boldsymbol{\theta} = [\,\Delta\rho_1 \quad \Delta\beta_1 \quad \Delta\rho_2 \quad \Delta\beta_2\,]^\top$$

where $\boldsymbol{\theta}^{(r)} = [\Delta\rho_r \quad \Delta\beta_r]^\top$, which features in (22), is the bias vector for sensor $r = 1, 2$. The covariance matrix in (22) is $\boldsymbol{\Sigma}_k^{(r)} = \mathrm{diag}[(\sigma_\rho^{(r)})^2, (\sigma_\beta^{(r)})^2]$. The probability of detection of sensor r is $p_\mathrm{D}^{(r)}$. The false detections are modeled by a Poisson point process, that is the intensity function $\kappa^{(r)}(\mathbf{z}|\boldsymbol{\theta})$ for sensor r, which features in (6), is $\kappa^{(r)}(\mathbf{z}|\boldsymbol{\theta}) = \mu_r s_r(\mathbf{z}|\boldsymbol{\theta})$, where μ_r is the mean number of false detections (at one time instant) and $s_r(\mathbf{z}|\boldsymbol{\theta})$ is their distribution over the measurement space $\mathcal{Z}$.

Each object state evolves in time according to the Markov transitional density, which is independent of the parameter vector $\boldsymbol{\theta}$. In particular, we adopt the nearly constant velocity model [36], that is:

$$\tau(\mathbf{x}|\mathbf{x}') = \mathcal{N}(\mathbf{x}; \mathbf{F}_k\mathbf{x}', \mathbf{Q}_k) \tag{24}$$

with $\mathbf{F}_k = \boldsymbol{\Phi}_k \otimes \mathbf{I}_2$, $\mathbf{Q}_k = \boldsymbol{\Omega}_k \otimes \mathbf{I}_2$, where $\otimes$ is the Kroneker product, $\mathbf{I}_2$ is the 2×2 identity matrix,

$$\boldsymbol{\Phi}_k = \begin{bmatrix} 1 & T_k \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{\Omega}_k = \psi \begin{bmatrix} \frac{T_k^3}{3} & \frac{T_k^2}{2} \\ \frac{T_k^2}{2} & T_k \end{bmatrix}, \tag{25}$$

T_k is the time interval between k and $k-1$ and ψ is process noise intensity.

B. Numerical Results

The considered scenario with a typical measurement set sequence $\mathbf{Z}_{1:K}$ is illustrated in Fig. 1(a). The total number of measurement sets available for estimation is $K = 15$. A varying number of moving targets is present in the scenario, with $n_k = 3$ for $k = 1, 2, 3$, $n_k = 5$ for $k = 4, \ldots, 12$ and $n_k = 4$ for $k = 13, 14, 15$. The initial state vectors of the three targets that exist from $k = 1$ onwards are: $[70\ \text{km}, -50\ \text{m/s}, -30\ \text{km}, 125\ \text{m/s}]^\top$, $[-80\ \text{km}, -100\ \text{m/s}, 130\ \text{km}, -125\ \text{m/s}]^\top$, and

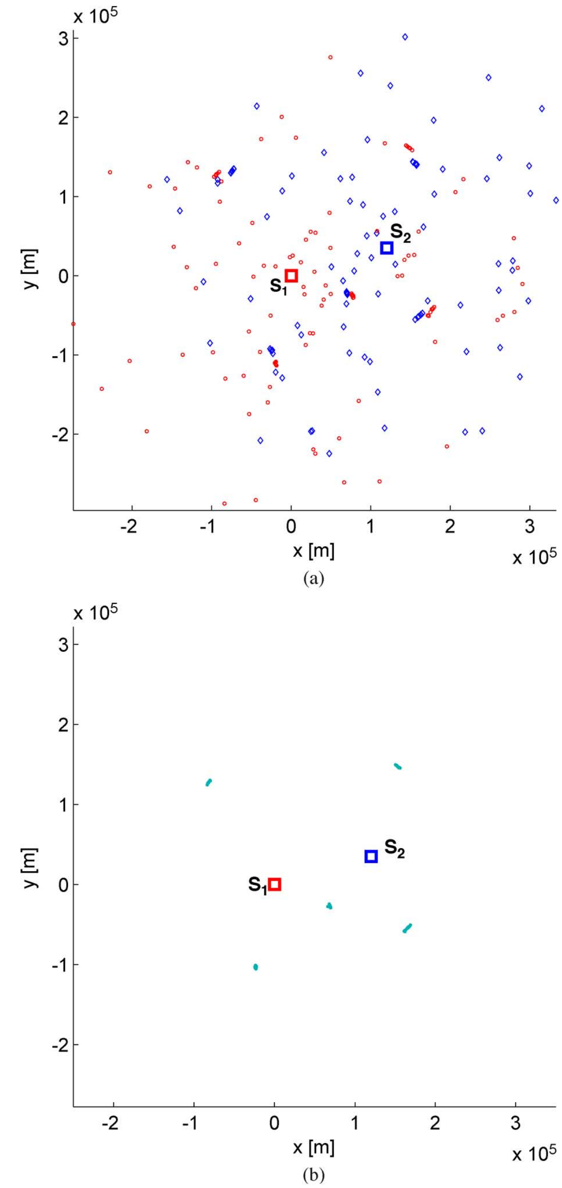

Fig. 1. The simulation setup: (a) the location of sensors (marked by □ and denoted S_1 and S_2) and the accumulated dataset $\mathbf{Z}_{1:K}$; (b) true target trajectories and estimated multi-target state $\widehat{\mathbf{X}}_{1:K}$ from the proposed algorithm (indistinguishable in this plot); note that the five target trajectories appear very short because they are observed over 45 seconds only.

$[170\ \text{km}, -200\ \text{m/s}, -50\ \text{km}, -200\ \text{m/s}]^\top$. The initial state vectors of the two targets that appear at $k = 4$ and exist onwards are: $[-25\ \text{km}, 50\ \text{m/s}, -100\ \text{km}, -165\ \text{m/s}]^\top$ and $[150\ \text{km}, 180\ \text{m/s}, 150\ \text{km}, -150\ \text{m/s}]^\top$. The time interval between k and $k-1$ is constant and equals $T_k = 3$ s. Target trajectories (estimated and true, they are indistinguishable) are shown in Fig. 1(b); they appear very short because the measurements are collected over 45 seconds only. The two sensors operate asynchronously; sensor 1 collects measurement sets at k =1, 3, 5, 7, 9, 11, 13, 15, while sensor 2 collects measurement sets at k = 2, 4, 6, 8, 10, 12, 14. With this sensor reporting setup, the multi-sensor PHD filter update reduces to the standard single-sensor case. The two sensors are placed at $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (120\ \text{km}, 35\ \text{km})$. The probability of detection is $p_\mathrm{D}^{(1)} = p_\mathrm{D}^{(2)} = 0.95$, the clutter parameters

TABLE I
CORRECTION FACTORS AND MCMC ACCEPTANCE RATES DURING THE PROGRESSIVE CORRECTION

s	1	2	3	4	5	6	7	8	9
λ_s	0.0229	0.0473	0.0768	0.1032	0.1308	0.1577	0.1845	0.2114	0.0653
Accept. rate (%)	9.3	20.7	20.2	22.8	11.9	12.2	5.3	6.9	33.6

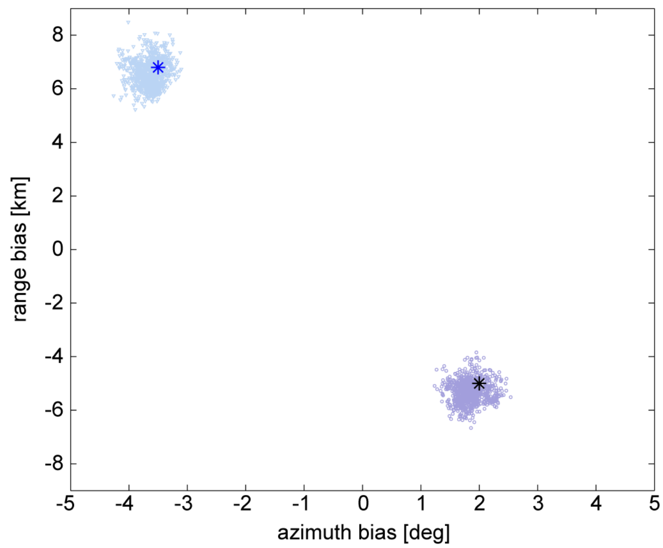

Fig. 2. A scattered plot of sensor bias particles at the end of progressive correction, $\{\boldsymbol{\theta}_j^S\}_{1\leq j\leq M}$, approximating the posterior $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$. True bias values indicated by asterisks.

are: $\mu_1 = \mu_2 = 10$, while $s_1(\mathbf{z}|\boldsymbol{\theta}) = s_2(\mathbf{z}|\boldsymbol{\theta})$ are uniform distributions over $\mathcal{Z} = [20\text{ km}, 300\text{ km}] \times [-\pi, \pi]$. The standard deviations of measurement noise are $\sigma_\rho^{(1)} = \sigma_\rho^{(2)} = 50$ km and $\sigma_\beta^{(1)} = \sigma_\beta^{(2)} = 0.5°$. The true values of sensor biases are set to $\Delta\rho_1 = 6.8$ km, $\Delta\beta_1 = -3.5°$, $\Delta\rho_2 = -5$ km, $\Delta\beta_2 = 2°$.

The particle PHD filter uses $L_k = 500$ and $N_k = 1000$ particles, with $p_S = 0.95$ and $n_b = 0.05$. The prior $p_0(\boldsymbol{\theta})$ is a uniform distribution over $[-10\text{ km}, +10\text{ km}] \times [-5°, +5°] \times [-10\text{ km}, +10\text{ km}] \times [-5°, +5°]$. The number of samples used in the bias space is $M = 1000$. The parameters used in adaptive selection of correction factors are: $\phi = 50$ and $H = 0.6$ M. This resulted in a sequence of correction factors listed in Table I.

The results of sensor bias estimation are shown in Fig. 2 (scattered plot) and Fig. 3 (marginalized histograms). Fig. 2 indicate the spread of the estimated posterior $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$. Note that the true bias values fall inside the two clouds of particles. The four figures in Fig. 3 display the (normalized) histograms of the output sample $\{\boldsymbol{\theta}_j^S\}_{1\leq j\leq M}$, which approximates the posterior $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$, marginalized to: (a) $\Delta\rho_1$, (b) $\Delta\beta_1$, (c) $\Delta\rho_2$, (d) $\Delta\beta_2$. The kernel density estimate (KDE) [37] of the corresponding marginalized density is also shown for each sensor bias (red solid line), as well as the true value of the bias (vertical dashed blue line). The maximum a posterior (MAP) estimates of sensor biases (computed from the KDE) are: $\Delta\rho_1 = 6.863$ km, $\Delta\beta_1 = -3.578°$, $\Delta\rho_2 = -5.104$ km, $\Delta\beta_2 = 1.806°$. While these estimates vary on every run of the algorithm, the support of the estimated posterior $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$ always contains the true bias values (as in Figs. 2 and 3). Fig. 1(b) shows an estimate of

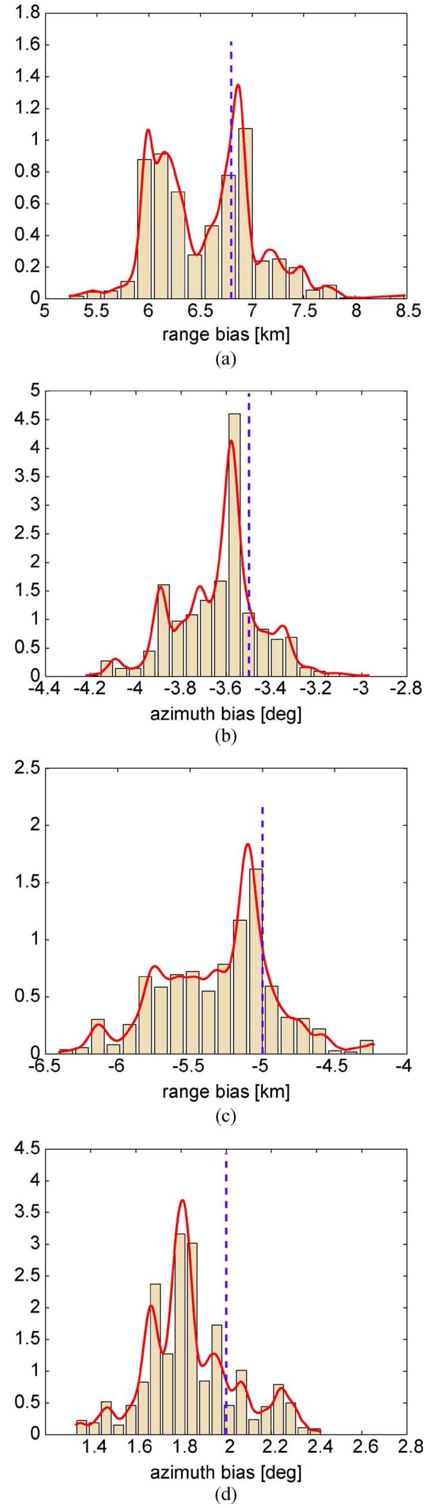

Fig. 3. Normalized histograms of the bias sample $\{\boldsymbol{\theta}_j^S\}_{1\leq j\leq M}$ approximating the posterior $p(\boldsymbol{\theta}|\mathbf{Z}_{1:K})$, marginalized to: (a) $\Delta\rho_1$, (b) $\Delta\beta_1$, (c) $\Delta\rho_2$, (d) $\Delta\beta_2$. The kernel density estimates shown by red solid lines; true values indicated by vertical blue dashed lines.

the multi-target state estimate $\widehat{\mathbf{X}}_{1:K}$, obtained using the MAP estimate of the sensor bias vector.

C. Discussion and Future Work

The empirical results of Section IV-B indicate remarkably accurate performance of the proposed algorithm even using a relatively small dataset. Some important questions, however, remain for future work. For example, while it is expected that observability of parameter $\boldsymbol{\theta}$ improves with the increase in the number of targets n_k (if $n_k = 0$, $\boldsymbol{\theta}$ cannot be estimated), a more rigorous study of observability remains elusive.

Since the particle PHD filter provides the estimate of the likelihood $f(\mathbf{Z}_{1:K}|\boldsymbol{\theta})$, the accuracy of the proposed calibration method clearly depends on the number of particles used in the particle PHD filter. In order to reduce the Monte Carlo variance of this estimate there is a need to use a reasonably large number of particles. This variability, however, can be also reduced by application of the backward pass through the data [18], as in the PHD smoother [38], [39]. Initial results for smoothing with the single-cluster PHD filter have been presented in [8].

The accuracy of parameter estimation also depends on the number of samples M in the parameter (bias) space, and the selection of the correction factors in the progressive correction. The results in Table I indicate a fairly low acceptance rate in this example. The MCMC acceptance rate can be increased using a lower value of parameter ϕ, which would result in smaller increments of the correction factors and consequently to an increase in the number of stages S.

Preliminary results of a recursive version of the proposed calibration technique have been presented in [9]. The recursive algorithm is important in situations where the calibration parameters are slowly varying. Its theoretical foundations follow from the single-cluster first-order moment filter derived in [4].

V. Conclusions

The paper presented a Monte Carlo method for static parameter estimation in multi-target tracking systems. The formulation is general and therefore applicable to calibration of any parameter in the target dynamic model and sensor measurement model, including target process noise level, environmental characteristics (clutter properties, propagation losses), or sensor parameters (biases, gains, detection probabilities).

The paper illustrated the proposed target system calibration algorithm in the context of multi-sensor translational-bias estimation. The outcome is a sensor bias estimation algorithm which is applicable to asynchronous sensors with imperfect detection, dynamic object appearance/disappearance and, most importantly, does not require the association of measurements to objects.

Appendix

Here we explain the step from (3) to (4). Let $f(\mathbf{X})$ be the multi-target density of a random finite set $\mathbf{X}$. Its corresponding PHD $D(\mathbf{x})$ can be obtained from [2, Eq.(.16.14)]

$$D(\mathbf{x}) = \int \delta_{\mathbf{X}}(\mathbf{x}) f(\mathbf{X}) \delta\mathbf{X} \tag{26}$$

where

$$\delta_{\mathbf{X}}(\mathbf{x}) = \sum_{\mathbf{w} \in \mathbf{X}} \delta_{\mathbf{w}}(\mathbf{x})$$

and $\delta_{\mathbf{w}}(\mathbf{x})$ is the standard Dirac delta concentrated at $\mathbf{w}$. The integral on the right-hand side of (26) is the *set integral*, defined as [2, p.361]:

$$\int g(\mathbf{X}) \delta\mathbf{X} = g(\emptyset) + \sum_{n=1}^{\infty} \frac{1}{n!} \int g(\{\mathbf{x}_1, \ldots, \mathbf{x}_n\})\, d\mathbf{x}_1 \cdots d\mathbf{x}_n$$

Using (26), the first moment of (3) with respect to $\mathbf{X}_{1:K}$ is given by:

$$\int \delta_{\mathbf{X}_{1:K}}(\mathbf{x}_{1:K}) f(\boldsymbol{\theta}, \mathbf{X}_{1:K}|\mathbf{Z}_{1:K})\, \delta\mathbf{X}_{1:K} \propto f(\mathbf{Z}_{1:K}|\boldsymbol{\theta})\, p_0(\boldsymbol{\theta}) \times \int \delta_{\mathbf{X}_{1:K}}(\mathbf{x}_{1:K}) f(\mathbf{X}_{1:K}|\mathbf{Z}_{1:K}, \boldsymbol{\theta})\, \delta\mathbf{X}_{1:K} \tag{27}$$

which can be written as (4).

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