Waveform Allocation for a MIMO Radar Network Using Potential Games

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***Abstract*—We propose a game theoretic waveform allocation algorithm for a MIMO radar network, where the radars are grouped into clusters. Using potential games, the clusters which are the players of the game, are able to choose a set of orthogonal waveforms from a set of available waveforms, that will result in optimum performance for the radars in the clusters. The simulation results demonstrate the superior performance of the game theoretic waveform selection as compared to a scheme that selects waveforms randomly.**

I. Introduction

Game theory is a branch of mathematics that provides the means to model, analyse and understand situations involving interactions among various decision makers [1]. In a radar network, where the radars operate in a non cooperative but at the same time non competitive environment, the goal of the individual radar is twofold: achieve the best possible detection performance and also operate in a way that is not competitive towards the other radars in the network.

The variety and flexibility of models in game theory make it a powerful tool for modelling situations involving radar systems. This is evident from the continuously increasing number of publications that use game theoretic schemes in radar applications in order to improve radar's performance. For example, the authors in [2] apply different polarisations on the transmission signal, and use zero-sum games to model the interaction between a distributed MIMO radar and an opponent, whose goal is to avoid being detected. Operating with different polarisations the radar aims to illuminate the most appropriate signal for maximising target detection. The interaction between a radar and an intelligent target equipped with a jammer is examined in [3]. As the goals of the two players are opposite, this interaction is modelled also as a zero-sum game. In [4], using the generalised Nash games, it is possible to allocate the optimum transmission power in a sensor radar network, such that the radars attain a specific target signal-to-disturbance ratio (SDR), while taking into account the interference induced in the network. The extension of this work to a MIMO radar network within the game theoretic framework is studied in [5] and [6]. Waveform design for a radar network is examined in [7], where using potential games the radars are able to use the appropriate waveform that will result in a maximum SDR.

In this work, we extend the work of [7] to a MIMO radar network which is partitioned into K clusters $C = \{C_1, \dots, C_K\}$. For all $k = 1, \dots, K$, cluster k consists of M radars, namely $C_k = \{R_{k1}, \dots, R_{kM}\}$. The aim of the clusters is to achieve good detection performance, measured in terms of SDR of the radars in each cluster. Motivated by the work in [7], using potential games [8], we model the interaction among the clusters as a game, where the players are the clusters and their strategies are different waveforms that the radars in the clusters can use to detect a target. Given a set of possible waveforms, the radars update their waveforms according to a game theoretic algorithm, until the scheme converges to an equilibrium. The equilibrium, is the state where the SDR of the radars is optimised by the appropriate choice of waveform. In other words, no player can profit by deviating from the equilibrium strategy, given the strategies of the other players.

II. Game Theoretic Formulation

A. Potential Games

Let $G =< \mathcal{N}, \{S_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}} >$ be a strategic form game, where $\mathcal{N}$ is a set of N players and S_i and u_i are the strategy set and the utility function of player i, respectively. Then, G is an exact potential or in short potential game, if and only if there exists a function $P : S \to \mathbb{R}$ such that for all $i \in \mathcal{N}$ and for all $\boldsymbol{s} \in S = S_1 \times \ldots \times S_N$

$$u_i(s_i, \boldsymbol{s}_{-i}) - u_i(s_i', \boldsymbol{s}_{-i}) = P(s_i, \boldsymbol{s}_{-i}) - P(s_i', \boldsymbol{s}_{-i}), \quad (1)$$

for all $s_i' \in S_i$, where $\boldsymbol{s}_{-i}$ is the vector of strategies of all players excluding player i. The function P is called the potential of the game. The equilibrium point is the strategy profile $\boldsymbol{s}^* \in S$ that maximises the potential function P ([8], Lemma 2.1). Additionally, notice that the function P depends only on the strategies of all players. Hence, we can think of the potential as a global function that reflects the change in utility from a unilateral change of a player's strategy. As a result, at the equilibrium point, the players not only maximise their individual payoffs, but they also maximise the overall welfare of the network.

B. Game Theoretic Model

Fig. 1 shows an example of a MIMO radar network with two clusters and two radar per cluster. In the network, we

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$$\text{SDR}_{kn} = \frac{G_{knkn}\sum_{r=1}^{M}|\alpha_{krn}|^2|\boldsymbol{s}_{kr}^H\boldsymbol{s}_{kr}|^2}{G_{knkn}\sum_{t=1}^{M}\sum_{r=1}^{M}\sum_{\substack{m=-N+1\\m\neq 0}}^{N-1}|\gamma_{ktn,m}|^2|\boldsymbol{s}_{kr}^H\boldsymbol{J}_m\boldsymbol{s}_{kt}|^2+\sum_{\substack{\ell=1\\\ell\neq k}}^{K}\sum_{t=1}^{M}G_{\ell tkn}\sum_{r=1}^{M}\sum_{m=-N+1}^{N-1}|\boldsymbol{s}_{kr}^H\boldsymbol{J}_m\boldsymbol{s}_{\ell t}|^2+\sigma_n^2\sum_{r=1}^{M}\boldsymbol{s}_{kr}^H\boldsymbol{s}_{kr}} \quad (2)$$

assume that communication among clusters is not feasible. However, radars within the same cluster can share information. We assume that the return signal to radar R_{kn} consists of N pulses, and radar R_{kn} processes the received signal using a bank of filters matched to the signature waveforms of all radars within the k^{th} cluster. Following the signal format of [7], the return signal for the radar R_{kn} can be written as

$$\boldsymbol{x}_{kn} = \sum_{r=1}^{M}\alpha_{krn}\boldsymbol{s}_{kr} + \sum_{\substack{\ell=1\\\ell\neq k}}^{K}\sum_{t=1}^{M}\sum_{m=-N+1}^{N-1}\boldsymbol{J}_m\boldsymbol{s}_{\ell t} + \sum_{r=1}^{M}\sum_{\substack{m=-N+1\\m\neq 0}}^{N-1}\gamma_{krn,m}\boldsymbol{J}_m\boldsymbol{s}_{kr} + \boldsymbol{n}_{kn},$$

where $\boldsymbol{s}_{kr}$ is the signal transmitted by the radar R_{kn}. The coefficients α_{krn} and $\gamma_{krn,m}$ account for the channel coefficients of the back-scattered signal from the target and any clutter from the environment, accordingly, associated with radar R_{kn}. Finally, $\boldsymbol{n}_{kn}$ denotes the noise induced within the radar R_{kn} and $\boldsymbol{J}_m$ is a shift matrix whose elements are given by

$$\boldsymbol{J}_m(i,j) = \boldsymbol{J}_{-m}(i,j) = \begin{cases} 1 & \text{if } j-i=m, \\ 0 & \text{otherwise.} \end{cases}$$

Following the notation of [7], let $G_{\ell tkn}$ be the antenna gain for the radar R_{kn} in the direction of radar $R_{\ell t}$ of the cluster $\ell \neq k$. Then, the SDR for radar R_{kn} is given by (2) at the top of the page. In the numerator we have the contribution of the return signals from all M radars at the receiver of the n^{th} radar in the k^{th} cluster. The waveform used by radar R_{kt} is denoted by $\boldsymbol{s}_{kt}$, and thus the first term on the denominator corresponds to the clutter returns in the direction of the target, due to illumination of all radars in the k^{th} cluster. The second term accounts for the interference induced by the radars from all other clusters. Finally, the third term in the denominator of (2) is the noise power for which we assumed $\sigma_n^2 = 1$.

The game for our model is defined by $\Pi = \langle C, \{S_k\}_{k\in\{1,\dots,K\}}, \{u_k\}_{k\in\{1,\dots,K\}}\rangle$, with C being the set of clusters in the network and S_k is the strategy set of player k. The strategy set is a collection of a finite number of tuples, where each tuple consists of M pairwise orthogonal waveforms. The utility function of player k is denoted by u_k. Furthermore, assuming that for all $k = 1,\dots,K$ and $n = 1,\dots,M$, the signals have norm $\|\boldsymbol{s}_{kn}\|_2^2 = 1$, the maximisation of the SDR is equivalent to the minimisation of the denominator. Extending the potential function of [7] to the case of a MIMO radar network and assuming for a moment that the radar cross section coefficients α_{krn} and $\gamma_{krn,m}$ are not part of the signal model, we define a function P to be the sum of the denominators from the SDR expression of all radars in the network:

$$\begin{aligned} P(\boldsymbol{s}_1,\dots,\boldsymbol{s}_K) = -\sum_{k=1}^{K}\sum_{n=1}^{M}\Bigg(&\sum_{r=1}^{M}\boldsymbol{s}_{kr}^H\boldsymbol{s}_{kr} \\ &+ G_{knkn}\sum_{t=1}^{M}\sum_{r=1}^{M}\sum_{\substack{m=-N+1\\m\neq 0}}^{N-1}|\boldsymbol{s}_{kr}^H\boldsymbol{J}_m\boldsymbol{s}_{kt}|^2 \\ &+ \sum_{\substack{\ell=1\\\ell\neq k}}^{K}\sum_{t=1}^{M}G_{\ell tkn}\sum_{r=1}^{M}\sum_{m=-N+1}^{N-1}|\boldsymbol{s}_{kr}^H\boldsymbol{J}_m\boldsymbol{s}_{\ell t}|^2\Bigg). \end{aligned} \quad (3)$$

The utility function for cluster k is the sum of the denominators of the SDR from the radars $R_{k1},\dots,R_{kM}$ in cluster k, together with the terms that correspond to the interference that these radars cause to all other radars in the network:

$$\begin{aligned} u_k(\boldsymbol{s}_1,\dots,\boldsymbol{s}_K) = -\sum_{n=1}^{M}\Bigg(&\sum_{r=1}^{M}\boldsymbol{s}_{kr}^H\boldsymbol{s}_{kr} \\ &+ G_{knkn}\sum_{t=1}^{M}\sum_{r=1}^{M}\sum_{\substack{m=-N+1\\m\neq 0}}^{N-1}|\boldsymbol{s}_{kr}^H\boldsymbol{J}_m\boldsymbol{s}_{kt}|^2 \\ &+ \sum_{\substack{\ell=1\\\ell\neq k}}^{K}\sum_{t=1}^{M}G_{\ell tkn}\sum_{r=1}^{M}\sum_{m=-N+1}^{N-1}|\boldsymbol{s}_{kr}^H\boldsymbol{J}_m\boldsymbol{s}_{\ell t}|^2 \\ &+ \sum_{\substack{\ell=1\\\ell\neq k}}^{K}\sum_{t=1}^{M}G_{kn\ell t}\sum_{r=1}^{M}\sum_{m=-N+1}^{N-1}|\boldsymbol{s}_{\ell r}^H\boldsymbol{J}_m\boldsymbol{s}_{kt}|^2\Bigg). \end{aligned} \quad (4)$$

A closer look at the utility function will show that the utility is included in (3). In particular, the utility of player k is formed of those parts in (3) that are associated with player k, namely the interference that all radars within cluster k experience from and cause to all radars in the other clusters in the network, and the clutter returns coming from the signal transmitted by the radars in the k^{th} cluster. Thus, it is straightforward to show that (1) is satisfied, and so P is a potential function and Π a potential game. According to [8], the equilibrium of G can be found by maximising the potential function with respect to the strategies (waveforms) of the players (clusters) *i.e.*

$$\max_{(\boldsymbol{s}_1,\dots,\boldsymbol{s}_K)\in S} P(\boldsymbol{s}_1,\dots,\boldsymbol{s}_K).$$

The clusters engage in an iterative process. In a sequential manner, they update their waveforms according to the above maximisation problem, until the game theoretic algorithm

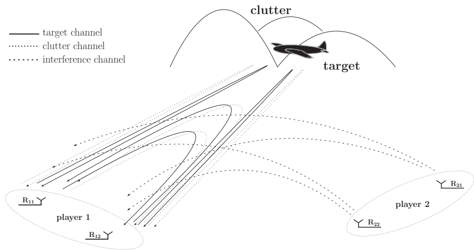

Fig. 1. Example of a MIMO radar network with two clusters and two radars in each cluster.

converges to an equilibrium. At each time step, only one cluster updates the waveform. Due to the particular connection between the potential function and the utility of player k, when it is his turn to update the waveform, instead of maximising the potential function, he can maximise the utility function, since for all $k = 1, \ldots, K$, u_k is incorporated in P. The converge of this iterative process is guaranteed due to the finite nature of the game and a result in [8] proving the existence of equilibrium for finite potential games.

Assuming that the waveform library from which each cluster can choose his strategy set, the antenna gains and the initial set of waveforms in the game are publicly known, then each cluster can independently recreate the game and arrive to the same equilibrium, which will be the actual transmitted signal. However, the radar cross section coefficients are part of the transmission-reception process of each radar in the network, and thus can not be determined beforehand and be available to all clusters. For this reason, they are not considered as part of our game. However, in order to demonstrate the value of the game theoretic approach, we show through simulations that this assumption does not undermine the detection performance of the radars.

TABLE I
ANTENNA GAINS IN DB FOR TWO DIFFERENT NETWORK CONFIGURATIONS.

Network with $K = 2,\ M = 2$
$(G_{1111}, G_{2111}, G_{2211}) = (0, -30, -16)$
$(G_{1212}, G_{2112}, G_{2212}) = (0, -27, -25)$
$(G_{2121}, G_{1121}, G_{1221}) = (0, -13, -14)$
$(G_{2222}, G_{1122}, G_{1222}) = (0, -17, -28)$
Network with $K = 3,\ M = 2$
$(G_{1111}, G_{2111}, G_{2211}, G_{3111}, G_{3211}) = (0, -30, -16, -11, -21)$
$(G_{1212}, G_{2112}, G_{2212}, G_{3112}, G_{3212}) = (0, -27, -25, -27, -16)$
$(G_{2121}, G_{1121}, G_{1221}, G_{3121}, G_{3221}) = (0, -13, -14, -28, -28)$
$(G_{2222}, G_{1122}, G_{1222}, G_{3222}, G_{3222}) = (0, -17, -28, -26, -11)$
$(G_{3131}, G_{1131}, G_{1231}, G_{2131}, G_{2231}) = (0, -19, -27, -13, -29)$
$(G_{3232}, G_{1132}, G_{1232}, G_{2132}, G_{2232}) = (0, -16, -11, -23, -18)$

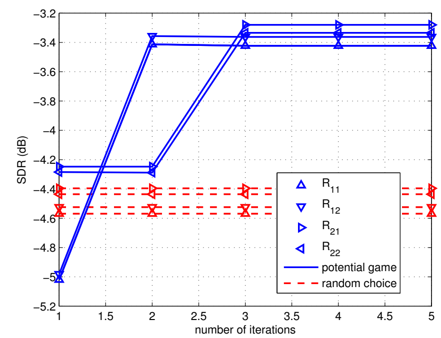

Fig. 2. SDR values for all radars using the game theoretic model and the random model. The network consists of two clusters with two radars per cluster.

III. SIMULATION RESULTS

For the simulation, we considered two network topologies. In the first case, the MIMO radar network consists of two clusters with two radars each $(K = 2,\ M = 2)$, while in the second case, we have three clusters with two radars each $(K = 3,\ M = 2)$. The initial waveforms for all clusters are chosen randomly from a set of possible waveforms. As the radars in each cluster form a MIMO configuration, this set was created by finding pairs of orthogonal waveforms in the waveform library described in [7]. The waveforms that are used by radars from different clusters might be correlated. The antenna gains in dB for the two network topologies were set as in Table I.

We assume that any clutter echoes result from clutter that is situated in the direction of the target. Hence, the antenna gains $G_{1112}, G_{1211}, G_{2122}, G_{2221}, G_{3132}$ and G_{3231} are ignored. Fig. 2 and Fig. 3 show the convergence of the game theoretic algorithm to an equilibrium (solid line) for the two network configurations. In Fig. 2, the network consists of two clusters with two radars in each cluster, while in Fig. 3 we have three clusters with two radars per cluster. The sequential update of the waveform for each player is clearly depicted. In order to demonstrate the advantages of our game theoretic model, both Fig. 2 and Fig. 3 show also the SDR for the players when they choose the waveforms randomly (random choice model). For each player in the random choice model, the SDR is the average over 100 realisations. The game theoretic waveform selection provides substantially better SDR as compared to selection of waveforms randomly. Fig.4 shows how the SDR of player one progresses with the increase of the network size. This evidences that the good performance of the game theoretic model is preserved, independently of the size of the network.

In all the above simulations the SDR was calculated without considering the effect of the coefficients $|\alpha_{krn}|^2$ and $|\gamma_{krn,m}|^2$, which were set to 1 in (2). To address the matter of the radar cross section coefficients, Table II presents the SDR values calculated using (2) for both the game theoretic and the random choice models, for the two network topologies. We chose the coefficients $|\alpha_{krn}|^2$ and $|\gamma_{krn,m}|^2$ such that they follow the Rayleigh and Weibull distributions, respectively. The parameter for the Rayleigh distribution is $a = 2.7$, while

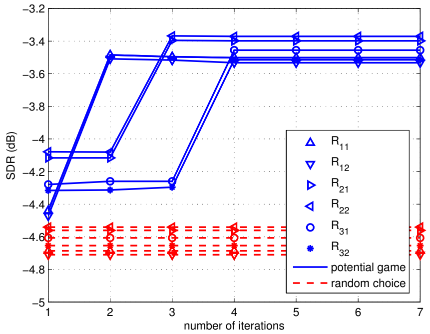

Fig. 3. SDR values for all radars using the game theoretic model and the random model. The network consists of three clusters with two radars per cluster.

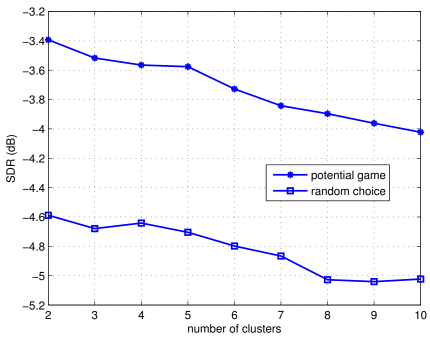

Fig. 4. Average of the two SDR values of the radars R_{11} and R_{12} in the game theoretic model and the random choice model for a network with increasing number of clusters. All clusters in the network consist of two radars.

for the Weibull we set $a = 1.1$ and $b = 2.6$. Calculating the SDR for 100 different values of $|\alpha_{krn}|^2$ and $|\gamma_{krn,m}|^2$, Table II presents the average SDR for the two models. Notice, that the waveforms used in the calculation of SDR for the potential game was the waveform at equilibrium. The results show that the game theoretic approach is preferable over the random selection of waveform.

Finally, Fig. 5 presents similar results to Fig. 4, with the difference that the SDR is calculated and then averaged over 1000 instances with different radar cross section coefficients that follow the aforementioned distributions. The cluster's performance when using the game theoretic model is evidently better in comparison to the random choice model, even in the presence of great interference caused by nine other clusters in the network.

IV. Conclusion

We have proposed a game theoretic scheme for waveform allocation in a MIMO radar network. In particular, we used potential games to model the interaction between the MIMO radars of the network and to choose appropriate waveforms from a waveform library, that results in a good SDR for the MIMO clusters. Simulation results demonstrated convergence to the equilibrium of the game and superior performance as compared to the random selection of waveforms.

TABLE II
AVERAGE SDR FOR THE TWO NETWORK TOPOLOGIES, EVALUATED AT WAVEFORMS OBTAINED USING THE GAME THEORETIC ALGORITHM AND WAVEFORMS CHOSEN AT RANDOM. THE SDR HAS BEEN CALCULATED TAKING INTO ACCOUNT THE RADAR CROSS SECTION COEFFICIENTS.

Average SDR: $K = 2, R = 2$

	Radar	**Potential Game**	**Random Choice**
Player 1	(R_{11}, R_{12})	(2.0529, 1.8821)	(0.9448, 0.7156)
Player 2	(R_{21}, R_{22})	(2.0700, 1.5869)	(0.9283, 0.4574)

Average SDR: $K = 3, R = 2$

	Radar	**Potential Game**	**Random Choice**
Player 1	(R_{11}, R_{12})	(1.9087, 2.1030)	(0.7968, 0.9533)
Player 2	(R_{21}, R_{22})	(2.2041, 1.6529)	(1.0950, 0.4851)
Player 3	(R_{31}, R_{32})	(2.1522, 2.5003)	(1.0138, 1.3580)

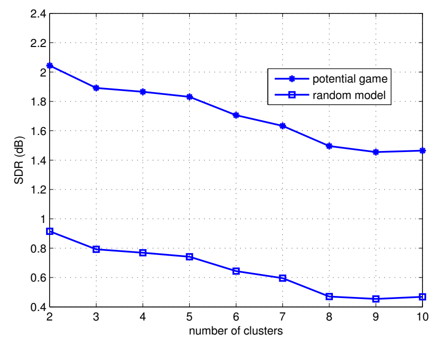

Fig. 5. SDR values for the cluster C_1, averaged first, over 1000 instances with different radar cross section coefficients, and then over the two radars R_{11} and R_{12}. in the game theoretic model and the random choice model for a network with increasing number of clusters. All clusters in the network consist of two radars.

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