GAME THEORETIC POWER ALLOCATION TECHNIQUE FOR A MIMO RADAR NETWORK

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ABSTRACT

We propose a waveform power allocation technique for a set of clusters of radars in a radar network using a game theoretic method. Each cluster consists of a number of radars forming a MIMO configuration. There is no communication between the clusters in the network, hence the power allocation in the radars within each cluster is performed using a non-cooperative game theoretic technique. The aim of each cluster of radars in the network is to minimise the total power used by the radars in its cluster while achieving a target detection criterion. The convergence of the algorithm to the Nash equilibrium is demonstrated.

Index Terms— MIMO radar, power allocation, distributed radar, game theory

1. INTRODUCTION

Radar networks with distributed transmitters and receivers have many advantages in terms of providing diversity and enhancing detection performance. However, the operation of such a network requires good coordination for mitigating undesirable interference. Decentralised resource allocation algorithms with minimal coordination between the radars are important to exploit fully the potential of such radar networks. Recently, a game theoretic method has been proposed in [1] for controlling the transmission power of the radars in the network. Motivated by this work, we consider the problem of power allocation in a radar network, but in contrast to [1] we assume a network that consists of groups of Multiple-Input Multiple-Output (MIMO) radars, with either collocated or widely separated antennas. MIMO refers to a radar architecture that uses multiple antennas for both transmission and reception [2]. Through the transmission of independent waveforms, MIMO radars offer spatial diversity of the target's cross section, increase the number of targets that can be identified (parameter identifiability), as well as detecting slow moving targets, by using the Doppler information from different directions.

Game theory has recently been widely used to model various radar situations. For example, in [3] a technique known as potential game was proposed for waveform design in radar networks. In [4] the authors modelled the interaction between a radar and a smart target equipped with a jammer using a zero-sum game. Using the signal polarization properties, the work in [5] proposed a polarimetric design scheme for distributed MIMO radar detection, also using zero-sum games.

In this paper we use game theoretic methods, and in particular non-cooperative games due to the absence of communication among the radar groups. The use of game theory offers the freedom to every MIMO radar group to act independently, and taking into account the actions of the other radar groups, to adjust the power in order to achieve its goals.

2. PROBLEM FORMULATION

We consider a radar network $C = \{C_1, \ldots, C_K\}$ that consists of K clusters, where each cluster is formed by M MIMO radars. Let us denote the set of radars that belong to the k^{th} cluster as $C_k = \{R_{k1}, \ldots, R_{kM}\}$. The underlying task of the clusters which operate under power constraints, is target detection. The aim of each cluster is to use the minimum possible transmission power in generating its waveforms in order to attain a specific detection performance, which is translated into achieving a certain signal-to-disturbance ratio (SDR). The clusters act independently of each other (non-cooperative behaviour), but do not compete with each other. This means that the clusters have no intention of deliberately interfering with other clusters. Such a situation can occur among a group of radars that belong to the same organisation. The groups share the same goal, but due to the difficulties of having communication among them, cooperation is infeasible. Figure 1 illustrates a typical MIMO radar network and associated parameters. At each time step, the radars receive N signal return samples and each radar makes the decision on

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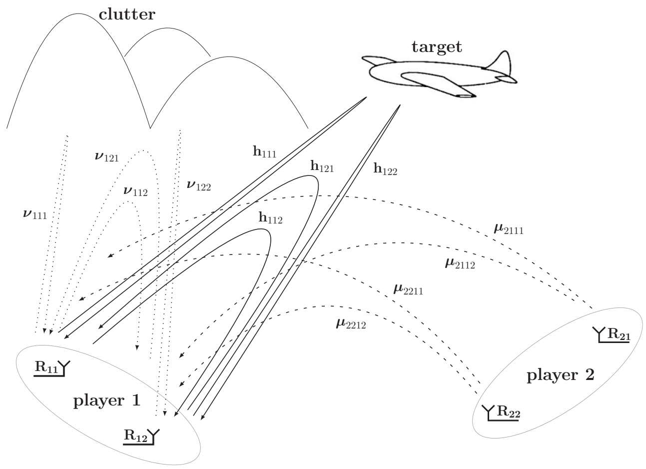


Fig. 1. A radar network with two MIMO radar clusters of size two and the signals that correspond to the radars of player 1.

the presence of a target by performing the hypothesis testing described below. In the following, $||\cdot||$ denotes the Euclidean norm and $\mathbf{v}^T$ is the transpose of a vector $\mathbf{v}$. We assume that the transmitted signals from radars within the same cluster are known to all radars in the cluster, and moreover, these transmitted signals are orthogonal to each other in order to exploit waveform diversity of the MIMO configuration. However, the waveforms of the radars that belong to different clusters may not be orthogonal. Using the notation introduced in [1] , let $\alpha_{kji} \sim \mathcal{CN}(0, h_{kji}p_{kj})$ be the gain from the j^{th} radar to the i^{th} radar in cluster k, h_{ki} and p_{ki} are signal propagation loss and transmission power, respectively, as shown in Figure 1. Also, $\beta_{\ell jki} \sim \mathcal{CN}(0, \mu_{\ell jki}p_{\ell j})$ denotes the cross channel gain between the i^{th} radar in cluster k and j^{th} radar in cluster ℓ. Then, the interference that the radar R_{ki} experiences from the other clusters is given by

$$\mathbf{i}_{ki} = \sum_{\substack{\ell=1\\ \ell\neq k}}^{K} \sum_{j=1}^{M} \beta_{\ell jki}\, \mathbf{s}_{\ell j}.$$

Finally, let the noise and the signal echoes from the clutters be denoted by $\mathbf{d}_{ki} \sim \mathcal{CN}(0, \nu_{kji}p_{kj} + \sigma_n^2)$, where $\nu_{kji}p_{kj}$ is the clutter power induced by all radars in the k^{th} cluster to the radar R_{ki} and σ_n^2 is the noise power. Using the above definitions, and for $k = 1\ldots, K$ and $i, j = 1, \ldots, M$, the hypothesis testing is formed as:

$$\mathcal{H}_0: \quad \mathbf{x}_{ki} = \mathbf{i}_{ki} + \mathbf{d}_{ki} \qquad \text{(target absent)}$$
$$\mathcal{H}_1: \quad \mathbf{x}_{ki} = \textstyle\sum_{j=1}^{M} \alpha_{kji}\mathbf{s}_{kj} + \mathbf{i}_{ki} + \mathbf{d}_{ki} \qquad \text{(target present)}$$

In order to determine the appropriate detector we apply the generalized likelihood ratio test (GLRT). Define $\boldsymbol{\alpha}_{ki} = [\alpha_{k1i}, \ldots, \alpha_{kMi}]^T$. Let

$$\phi_{\mathcal{H}_0}(\mathbf{x}_{ki}; \sigma^2_{\mathcal{H}_0}) = \frac{1}{(2\pi)^{N/2}\sigma^N_{\mathcal{H}_0}}\, e^{-\frac{||\mathbf{x}_{ki}||^2}{2\sigma^2_{\mathcal{H}_0}}} \tag{1}$$

$$\phi_{\mathcal{H}_1}(\mathbf{x}_{ki}; \boldsymbol{\alpha}_{ki}, \sigma^2_{\mathcal{H}_1}) = \frac{1}{(2\pi)^{N/2}\sigma^N_{\mathcal{H}_1}}\, e^{-\frac{||\mathbf{x}_{ki} - \sum_{j=1}^{M}\alpha_{kji}\mathbf{s}_{ki}||^2}{2\sigma^2_{\mathcal{H}_1}}} \tag{2}$$

denote the probability density functions of $\mathbf{x}_{ki}$ under hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. The maximum likelihood estimate for $\sigma^2_{\mathcal{H}_0}$ under $\mathcal{H}_0$ is $\hat{\sigma}^2_{\mathcal{H}_0} = ||\mathbf{x}_{ki}||^2/N$. Fixing $\sigma^2_{\mathcal{H}_1}$, the maximum likelihood estimate for α_{kji}, for all $i = 1, \ldots, M$ is obtained as $\hat{\alpha}_{kji} = \mathbf{s}^H_{kj}\mathbf{x}_{ki}/N$. Substituting $\hat{\alpha}_{kji}$ back in (1) and maximizing $f_{\mathcal{H}_1}(\mathbf{x}_{ki}; \boldsymbol{\alpha}_{ki}, \sigma^2_{\mathcal{H}_1})$ with respect to $\sigma^2_{\mathcal{H}_1}$ we obtain

$$\hat{\sigma}^2_{\mathcal{H}_1} = \frac{||\mathbf{x}_{ki} - \sum_{j=1}^{M}\alpha_{kji}\mathbf{s}_{ki}||^2}{N}$$

Finally, if $\lambda_{ki} \in [0, 1]$ is the detection threshold for each radar $i = 1, \ldots, M$ in cluster k, then the GLRT has the form

$$\frac{\phi_{\mathcal{H}_1}}{\phi_{\mathcal{H}_0}} = \frac{\sum_{j=1}^{M}|\alpha_{kji}\mathbf{s}_{ki}|^2}{||\mathbf{x}_{ki}||^2 N} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda_{ki}.$$

Furthermore, assuming a bank of M waveform matched filters is used in each radar, we define the SDR at the i^{th} radar in the k^{th} cluster as

$$\text{SDR}_{ki} = \frac{\sum_{j=1}^{M} h_{kji}p_{kj}}{\sum_{j=1}^{M}\nu_{kji}p_{kj} + \sum_{\substack{\ell=1\\ \ell\neq k}}^{K}\sum_{j=1}^{M}\mu_{\ell jki}p_{\ell j} + \sigma_n^2} \tag{3}$$

In order to measure the performance of the detection test we employ the probabilities of miss-detection P_{md} and false alarm P_{fa} for each radar. The calculation of the probabilities can be found in [6, 7]

$$P_{md}(\lambda_{ki}) = (1 - \lambda_{ki})^{N-1}$$
$$P_{fa}(\text{SDR}_{ki}, \lambda_{ki}) = 1 - (1 - \tfrac{\lambda_{ki}}{1-\lambda_{ki}}\tfrac{1}{1+N\,\text{SDR}_{ki}})^{1-N}$$

Let ε_{ki} be a design parameter that provides an upper bound on P_{md} and P_{fa}. We require that the sum of the two probabilities is below ε_{ki}:

$$P_{md}(\lambda_{ki}) + P_{fa}(\text{SDR}_{ki}, \lambda_{ki}) \leq \varepsilon_{ki}. \tag{4}$$

Following the analysis of [1], by equating (4) to ε_{ki} we obtain the optimum λ_{ki}, which we will denote λ^*_{ki}. Then, using λ^*_{ki} we can determine the optimum SDR_{ki} for each radar, which we call γ^*_{ki}:

$$\gamma^*_{ki} = \min\{\text{SDR}_{ki} \mid \exists\lambda_{ki} \in [0, 1] \\ \text{s.t. } P_{md}(\lambda_{ki}) + P_{fa}(\text{SDR}_{ki}, \lambda_{ki}) \leq \varepsilon_{ki}\}. \tag{5}$$

2.1. Game theoretical model

In order to examine the interaction among the radars and determine the best strategy for each cluster of radars, we model the scenario described above using game theory. Game theory offers the advantage for every MIMO cluster to act in a decentralised manner. In general, a game is defined as the tuple $< N, \{\mathcal{A}_i\}_{i\in N}, \{u_i\}_{i\in N} >$, where N is the set of players, and A_i and u_i are the set of actions and the payoff function accordingly, that are associated with each player. The solution concept that we will use is the Nash equilibrium, and is defined as the set of actions $(a_1^*, \ldots, a_N^*) \in \mathcal{A}_1 \times \ldots \times \mathcal{A}_N$ such that for all $i = 1, \ldots, N$

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i', a_{-i}^*), \quad \forall a_i' \in \mathcal{A}_i,$$

where the subscript $-i$ denotes all players excluding player i [8, 9]. In our case, the game is played among the MIMO clusters of the network. Let $C = \{C_1, \ldots, C_K\}$ be the set of players and define the action set for player $k \in \{1, \ldots, K\}$ to be:

$$\mathcal{P}_k(\mathbf{p}_{-k}) = \{\mathbf{p}_k \in [\underline{\mathbf{p}}_k, \overline{\mathbf{p}}_k] \subset \mathbb{R}_+^M \mid \mathrm{SDR}_{ki} \geq \gamma_{ki}, \forall i\}$$

where γ_{ki} is a fixed value and $\mathbf{p}_k = [p_{k1}, \ldots, p_{kM}]^T$ is the power allocated to radars in the cluster k, and $\underline{\mathbf{p}}_k$ and $\overline{\mathbf{p}}_k$ are the minimum and maximum possible values for $\mathbf{p}_k$, respectively. The utility function is then defined as

$$u_k(\mathbf{p}_k, \mathbf{p}_{-k}) = \sum_{i=1}^{M} p_{ki}$$

where $\mathbf{p}_{-k}$ denotes the power vector used by all other clusters except cluster k. Even though not shown explicitly, the power consumed by the k^{th} cluster, hence its utility $u_k(\mathbf{p}_k, \mathbf{p}_{-k})$, depends on the power consumed by all radars in other clusters through the constraint in (7). The underlying game is summarised as

$$\mathcal{G} =< C, \{\mathcal{P}_k\}_{k\in\{1,\ldots,K\}}, \{u_k\}_{k\in\{1,\ldots,K\}} > .$$

The best response strategy for the k^{th} player is the solution of the following optimisation:

$$\min_{\mathbf{p}_k \in \mathcal{P}_k(\mathbf{p}_{-k})} u_k(\mathbf{p}_k, \mathbf{p}_{-k}) \tag{6}$$

$$\text{s.t.} \quad \mathrm{SDR}_{ki}(\mathbf{p}_k, \mathbf{p}_{-k}) \geq \gamma_{ki}, \ \forall i = 1, \ldots, M \tag{7}$$

From the above formulation it is clear that the actions of the players are interdependent, since each player's SDR depends on the actions $\mathbf{p}_{-k}$ of the other players.

Let $S_k(\mathbf{p}_{-k})$ denote the interference plus noise term in SDR_{ki}, i.e. the last two terms in the denominator of (3). Then, solving (3) for $S_k(\mathbf{p}_{-k})$, we obtain

$$S_k(\mathbf{p}_{-k}) = \hat{H}_k \mathbf{p}_{-k}$$

	maximum power	$\overline{p}_{ki} = 1$	$\overline{p}_{ki} = 0.15$	$\overline{p}_{ki} = 0.12$
Custer 1	(p_{11}^*, p_{12}^*)	(0, 0.1744)	(0.0408, 0.15)	(0.0927, 0.12)
	total power	0.1744	0.1908	0.2127
Custer 2	(p_{21}^*, p_{22}^*)	(0.1593, 0)	(0.15, 0.0151)	(0.12, 0.0634)
	total power	0.1593	0.1651	0.1834

Table 1. Total allocated power $(k, i = 1, 2)$.

where

$$\hat{H}_k = \begin{bmatrix} \frac{h_{k11} - \hat{\gamma}_{k1}\nu_{k11}}{\hat{\gamma}_{k1}} & \cdots & \frac{h_{kM1} - \hat{\gamma}_{k1}\nu_{kM1}}{\hat{\gamma}_{k1}} \\ \vdots & \ddots & \vdots \\ \frac{h_{k1M} - \hat{\gamma}_{kM}\nu_{k1M}}{\hat{\gamma}_{kM}} & \cdots & \frac{h_{kMM} - \hat{\gamma}_{kM}\nu_{kMM}}{\hat{\gamma}_{kM}} \end{bmatrix}$$

and $\hat{\gamma}_{ki}$ is the instantaneous SDR of R_{ki}. The optimal solution of (6), (7) is $\mathbf{p}_k^* = H_k^{-1} S_k(\mathbf{p}_{-k})$, where

$$H_k = \begin{bmatrix} \frac{h_{k11} - \gamma_{k1}^*\nu_{k11}}{\gamma_{k1}^*} & \cdots & \frac{h_{kM1} - \gamma_{k1}^*\nu_{kM1}}{\gamma_{k1}^*} \\ \vdots & \ddots & \vdots \\ \frac{h_{k1M} - \gamma_{kM}^*\nu_{k1M}}{\gamma_{kM}^*} & \cdots & \frac{h_{kMM} - \gamma_{kM}^*\nu_{kMM}}{\gamma_{kM}^*} \end{bmatrix}.$$

3. EXPERIMENTAL RESULTS

In order to verify the convergence of the proposed algorithm using simulation results, we considered a network of four radars, which is partitioned into two clusters of size two. In every time step, upon receiving N signal samples, each radar updates its power in such a way that the overall power of the MIMO cluster is minimized until an equilibrium is reached. According to Theorem 2 of [10] the iterative algorithm should converge to a unique fixed point for any initial power values.

The number of received pulses is $N = 32$ and the Doppler shift is $f_{D,ik} = 0.1$ for all $k = 1, \ldots, K$ and $i = 1, \ldots, M$. We set the maximum number of iterations at $T = 100$. However, as seen in the simulation results, the algorithm converges within five iterations. Before the beginning of the game, the radars compute the optimum SDR value using (5), which is found to be $\gamma_{ki}^* = 3.76$ for all radars. The corresponding λ_{ki}^* is equal to 0.14 for all k, i. We set $\varepsilon_{ki} = 0.05$ (the upper bound on the sum of the probabilities P_{md} and P_{fa}). For a fixed known channel h_{kji}, the values of ν_{kji} and $\mu_{\ell jki}$ are set as $h_{kji}/10$ and $h_{kji}/20$, respectively, identical for all radars. Finally, the noise power is set to $\sigma_n^2 = 0.01$.

Figure 2 depicts the convergence of the game theoretic algorithm to an equilibrium point. The minimum and maximum possible power levels for each radar were set to 0.1W and 1W, respectively. At the equilibrium, we observe only one of the radar in each cluster is active and using positive power while the other radar is not illuminating any signal. This enables every radar in the cluster to take advantage of the signal transmitted by the active radar while keeping the interference in the network at low level due to inactiveness of the remaining radars. The power used by each radar in each cluster is shown in Table 1. We now set the maximum available power for radars in each cluster at 0.15 and then at 0.12. The convergence of the power values is shown in Figures 3(a) and 3(b), respectively. In the first case where the maximum power limit is set to 0.15, even though both radars in each cluster are in operation, the power is not allocated uniformly between the radars. One of the radars in the cluster uses most of the power, while the second is allocated with necessary additional power in order to attain the required SDR. The values of the power convergence points (p_{11}^*, p_{12}^*) and (p_{21}^*, p_{22}^*) are shown in Table 1. A similar scenario is illustrated in Figure 3(b), where the maximum power is limited to 0.12. Here, however, the power is allocated in a more balanced way between the radars in each cluster. This is because the maximum possible power for each radar is low enough to result in each radar using a similar amount of power. However, it should be highlighted that when there is no upper limit constraint on the power, the sum power required is lower than that when the constraints are imposed.

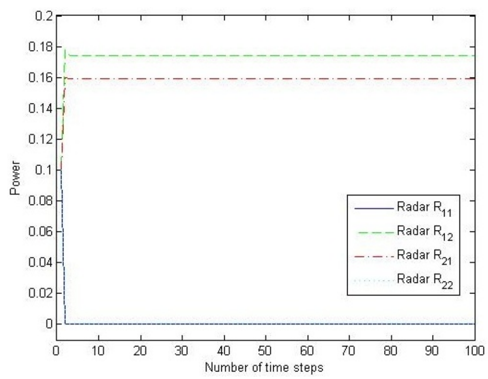
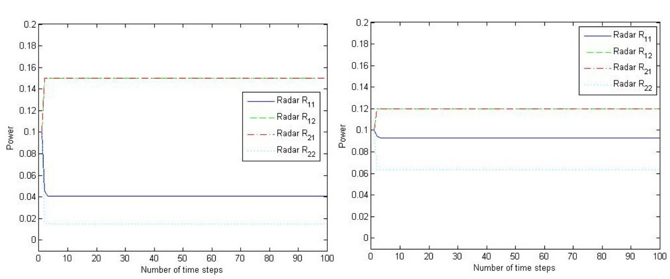


Fig. 2. Power allocation with adequate available power for both clusters: $[\underline{p}_{ki}, \overline{p}_{ki}] = [0.1, 1], \forall k, i$.

4. CONCLUDING REMARKS

We have proposed a game theoretic algorithm for power allocation in a MIMO radar network. The simulation confirmed



(a) (b)

Fig. 3. Power allocation with limited maximum power (a) $\overline{p}_{1i} = \overline{p}_{2i} = 0.15, \forall i$ (b) $\overline{p}_{1i} = \overline{p}_{2i} = 0.12, \forall i$.

the algorithm converges to a stable equilibrium. For the case of no constraints on the maximum power limit, only one of the radars in each cluster aims to illuminate signals which can be used by the peer radars in the cluster as a signal of opportunity for detection. This keeps the overall interference in the network low. However, when there is a power limit, the radars in each cluster share the transmission power in order to achieve a desired signal-to-disturbance ratio. The future work will consider joint power allocation, waveform design and beamforming using game theoretic methods.

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