Game theoretic power allocation for a multistatic radar network in the presence of estimation error

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***Abstract*—We investigate distributed power allocation techniques for a multistatic multi-input and multi-output (MIMO) radar network based on a game theoretic framework. We consider a network of radars grouped into clusters that aim to attain a specific signal-to-disturbance ratio (SDR) while using minimum possible total transmission power. We consider game theoretic power adaptation based on the estimate of SDR, so that there is no communication requirement between radars in the clusters. We demonstrate equilibrium convergence of the non-cooperative game theoretic algorithm even in the presence of estimation error for the SDR.**

I. Introduction

Game theoretic methods have recently been widely studied within the context of radars and as a result, various schemes have been developed that optimise the radars' transmission parameters according to the underlying scenarios. In particular, zero-sum games are used in [1] for the design of polarimetric waveforms that best capture the characteristics of the target, in order to maximise the detection performance. In [2] a zero-sum game theoretic approach has been used to investigate the interaction between a MIMO radar and an intelligent target featuring a jammer. The authors in [3] propose a code optimization technique through potential games in a radar network. A radar network is also considered in [4] where a generalised Nash game is used to control the transmission power of the radars.

Motivated by the work in [4] we propose a game theoretic framework for optimising the transmission power in a MIMO radar network with the radars being partitioned into clusters [5]. The goal of each cluster of radars is to achieve a particular signal-to-disturbance ratio (SDR) with the minimum possible transmission power and without causing deliberate interference to the other clusters in the network. As there is no communication between clusters, the game theoretic algorithm requires estimation of the SDR. In this paper, we extend the SDR estimation technique proposed in [4] to a MIMO radar network and investigate the performance of the game theoretic power allocation technique.

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II. Model description

We examine the case where a network is formed of a number of clusters $C = \{C_1, \ldots, C_K\}$ each of which consists of M radars, i.e. $C_k = \{R_{k1}, \ldots, R_{kM}\}$ for all $k = 1, \ldots, K$. The radars in each cluster aim to detect a target using the minimum possible total transmission power. This means that every cluster seeks to achieve its own SDR, while the total power of the radars within each cluster is minimised.

We assume that communication and information sharing occurs among the radars in the same cluster, but is not feasible among the different clusters in the network. However, the environment is not competitive. This suggests that although not cooperating, the clusters should not cause interference to the network deliberately. Hence, their aim is to minimise their transmission power taking into account both the clutter in the environment and the induced interference.

To detect the presence of the target, each cluster performs binary hypothesis testing based on the the generalised likelihood ratio test (GLRT) that is proposed in [4], [5]. In the presence of a target, the return signal received by the radar R_{ki} is given by

$$\mathbf{x}_{ki} = \sum_{j=1}^{M} \alpha_{kji} \widetilde{\mathbf{s}}_{kj} + \mathbf{i}_{ki} + \mathbf{d}_{ki} \tag{1}$$

where $\widetilde{\mathbf{s}}_{kj} = \mathbf{s}_{kj} \odot \mathbf{f}_D$ is the Hadamard product of the transmitted signal $\mathbf{s}_{kj}$ and the Doppler shift $\mathbf{f}_D$ associated with the movement of the target. The parameter $\alpha_{kji} \sim \mathcal{CN}(0, h_{kji} p_{kj})$ describes the channel gain in the direction of the target coming from the radar R_{ki} of cluster k, for all $i = 1, \ldots, M$. The parameters h_{kji} and p_{kj} denote the average signal propagation loss and transmitted power, respectively. The term $\mathbf{i}_{ki}$ introduces the interference to radar R_{ki} due to illumination of signals by radars from all other clusters in the network and is given by

$$\mathbf{i}_{ki} = \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \sum_{j=1}^{M} \beta_{\ell jki} \, \mathbf{s}_{\ell j}$$

with $\beta_{\ell jki} \sim \mathcal{CN}(0, \mu_{\ell jki} p_{\ell j})$ denoting the cross-channel gain from the j^{th} radar in the ℓ^{th} cluster to the i^{th} radar of cluster k. Finally, the clutter and noise factors are incorporated into the term $\mathbf{d}_{ki} \sim \mathcal{CN}(0, \sum_{j=1}^{M} \nu_{kji} p_{kj} + \sigma_n^2)$, where $\nu_{kji} p_{kj}$

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represents the clutter power and σ_n^2 the noise power. Using the above definitions, the SDR for the i^{th} radar in the k^{th} cluster is written as

$$\text{SDR}_{ki} = \frac{\sum_{j=1}^{M} h_{kji} p_{kj}}{\sum_{j=1}^{M} \nu_{kji} p_{kj} + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \sum_{j=1}^{M} \mu_{\ell jki} p_{\ell j} + \sigma_n^2}. \quad (2)$$

III. Game theoretic formulation

The model presented above describes a scenario where depending on the inter-cluster interference, the radars in each cluster have to adjust their transmission power in order to achieve their target SDR. Increased interference from cluster k forces the radars in the other clusters to increase their transmission power, and this in turn induces more interference to the environment which is not desirable to any radars. Employing game theory, we model this situation as the following generalised Nash game [6]

$$G = < \mathcal{C}, (\mathcal{P}_k)_{k\in\{1,\dots,K\}}, (S_k)_{k\in\{1,\dots,K\}}, (u_k)_{k\in\{1,\dots,K\}} >,$$

where C is the set of clusters acting as players, $\mathcal{P}_k$ denotes the action set associated with each player, S_k is a point-to-set mapping that describes the strategy set and u_k is the utility function for each player. In the following we adopt the customary notation '$-k$' in the subscript to denote all players excluding player k. The action set of the k^{th} player is $\mathcal{P}_k = \mathcal{P}_{k1} \times \ldots \times \mathcal{P}_{kM}$ with

$$\mathcal{P}_{ki} = \{p_{ki} \in \mathbb{R}^+ \mid p_{ki} \in [\underline{p}_{ki}, \overline{p}_{ki}]\}, \quad \forall i \in \{1,\dots,M\}$$

where $\underline{p}_{ki}$ and $\overline{p}_{ki}$ denote the minimum and maximum available powers, respectively for radar R_{ki}. The strategy set is defined as $S_k : \mathcal{P}_{-k} \twoheadrightarrow \mathcal{P}_k$ with

$$S_k(\mathbf{p}_{-k}) = \{\mathbf{p}_k \in \mathcal{P}_k \mid \text{SDR}_{ki} \geq \gamma_k^*, \ \forall i = 1,\dots,M\}, \quad (3)$$

where γ_k^* is the target SDR for cluster k and it is calculated using the probabilities of false alarm P_{fa} and miss-detection P_{md}, as well as the threshold λ_k of the GLRT [4], [5] and a specific design parameter ε_k:

$$\gamma_k^* = \min\{\text{SDR}_k \mid \exists \lambda_k \in [0,1] \ \text{s.t.} \ P_{md}(\lambda_k) + P_{fa}(\text{SDR}_k, \lambda_k) \leq \varepsilon_k\}. \quad (4)$$

The interdependency of the strategies is clearly described through the SDR constraints (3), as both the transmission power of player k and the powers used by all other players appear in the SDR (2). Finally, we define the utility function of player k to be

$$u_k(\mathbf{p}_{-k}, \mathbf{p}_k) = \sum_{i=1}^{M} p_{ki}.$$

The Nash equilibrium for the game G is the strategy profile $(\mathbf{p}_{-k}^*, \mathbf{p}_k^*)$ for which $\mathbf{p}_k^* \in S_k(\mathbf{p}_{-k}^*)$ and

$$u_k(\mathbf{p}_{-k}^*, \mathbf{p}_k^*) = \min_{\mathbf{p}_k \in S_k(\mathbf{p}_{-k}^*)} u_k(\mathbf{p}_{-k}^*, \mathbf{p}_k).$$

Using the best response strategy

$$\text{BR}_k(\mathbf{p}_{-k}) = \{\mathbf{p}_k \in \mathcal{P}_k \mid u_i(\mathbf{p}_{-k}, \mathbf{p}_k) \leq u_i(\mathbf{p}_{-k}, \mathbf{p}_k'), \ \forall \mathbf{p}_k' \in \mathcal{P}_k\},$$

the equilibrium $(\mathbf{p}_{-k}^*, \mathbf{p}_k^*)$ is the strategy such that $\mathbf{p}_k^* \in \text{BR}_k(\mathbf{p}_{-k}^*)$. In other words, every player has to solve the following optimisation problem

$$\min_{\mathbf{p}_k \in \mathcal{P}_k} u_k(\mathbf{p}_{-k}, \mathbf{p}_k) \ \text{ s.t. } \ \text{SDR}_{ki} \geq \gamma_k^*, \ \forall i = 1,\dots,M \quad (5)$$

iteratively until the equilibrium is reached. From (2) it is clear that the calculation of SDR_{ki} in the above constraints requires the knowledge of the interference plus noise terms $\sum_{\substack{\ell=1 \\ \ell\neq k}}^{K} \sum_{j=1}^{M} \mu_{\ell jki} p_{\ell j} + \sigma_n^2$, which is not possible to be acquired without any communication between the radar R_{ki} and all radars from the other clusters in the network. As such communication is not assumed in our work, radar R_{ki} aims to calculate these unknown terms by using the estimate of instantaneous SDR, denoted by $\hat{\gamma}_{ki}$. The method for estimating $\hat{\gamma}_{ki}$ is described in the next section. Using $\hat{\gamma}_{ki}$ instead of SDR_{ki} in (2) and rearranging, we obtain

$$\sum_{\substack{\ell=1 \\ \ell\neq k}}^{K} \sum_{j=1}^{M} \mu_{\ell jki} p_{\ell j} + \sigma_n^2 = \sum_{j=1}^{M} \frac{(h_{kji} - \hat{\gamma}_{ki}\nu_{kji})}{\hat{\gamma}_{ki}} p_{kj},$$

where the terms in the right-hand side are known to radar R_{ki}, and can be written in a matrix form as

$$\begin{bmatrix} \frac{h_{k1i} - \hat{\gamma}_{ki}\nu_{k1i}}{\hat{\gamma}_{ki}} & \dots & \frac{h_{kMi} - \hat{\gamma}_{ki}\nu_{kMi}}{\hat{\gamma}_{ki}} \end{bmatrix} \begin{bmatrix} p_{k1} & \dots & p_{kM} \end{bmatrix}^T$$

The constraints in the optimization problem (5) include the calculation of the SDR for all M radars in the k^{th} cluster. Thus, following the above procedure and using the instantaneous SDR $\hat{\gamma}_{ki}$, for all $i = 1,\dots,M$ we can substitute the interference plus noise terms for all radars in cluster k by:

$$\begin{bmatrix} \frac{h_{k11} - \hat{\gamma}_{k1}\nu_{k11}}{\hat{\gamma}_{k1}} & \dots & \frac{h_{kM1} - \hat{\gamma}_{k1}\nu_{kM1}}{\hat{\gamma}_{k1}} \\ \vdots & \ddots & \vdots \\ \frac{h_{k1M} - \hat{\gamma}_{kM}\nu_{k1M}}{\hat{\gamma}_{kM}} & \dots & \frac{h_{kMM} - \hat{\gamma}_{kM}\nu_{kMM}}{\hat{\gamma}_{kM}} \end{bmatrix} \begin{bmatrix} p_{k1} \\ \vdots \\ p_{kM} \end{bmatrix} = \hat{H}_k \mathbf{p}_k$$

Additionally, using (2) with the target SDR γ_k^* instead of SDR_{ki}, the clutter and target terms are written in matrix form as

$$\begin{bmatrix} \frac{h_{k11} - \gamma_k^*\nu_{k11}}{\gamma_k^*} & \dots & \frac{h_{kM1} - \gamma_k^*\nu_{kM1}}{\gamma_k^*} \\ \vdots & \ddots & \vdots \\ \frac{h_{k1M} - \gamma_k^*\nu_{k1M}}{\gamma_k^*} & \dots & \frac{h_{kMM} - \gamma_k^*\nu_{kMM}}{\gamma_k^*} \end{bmatrix} \begin{bmatrix} p_{k1} \\ \vdots \\ p_{kM} \end{bmatrix} = H_k \mathbf{p}_k$$

Hence, at time t the radars in the k^{th} cluster update their power by solving the optimisation problem:

$$\min_{\mathbf{p}_k \in \mathcal{P}_k} u_k(\mathbf{p}_{-k}, \mathbf{p}_k) \quad \text{s.t.} \quad H_k \mathbf{p}_k^{(t)} \geq \hat{H}_k \mathbf{p}_k^{(t-1)} \quad (6)$$

until the algorithm converges to the equilibrium. The existence of the equilibrium is guaranteed through the theorem by Arrow-Debreu [6] on convexity.

IV. SDR ESTIMATION

At each time step, the radars receive N signal return samples and engage in an iterative process during which they solve the optimisation problem (6) that requires estimation of the instantaneous SDR $\hat{\gamma}_{ki}$. Direct calculation of $\hat{\gamma}_{ki}$ requires the knowledge of the power transmitted from the radars in the other clusters of the network. Since communication among the clusters is not possible, this information cannot be obtained. Following the model by [4] the radar R_{ki} can estimate $\hat{\gamma}_{ki}$ using the following:

$$\hat{\gamma}_{ki} = \frac{\frac{\sum_{j=1}^{M} |\tilde{\mathbf{s}}_{kj}^H \mathbf{x}_{ki}|^2}{N} - \frac{||\mathbf{x}_{ki}||^2}{N}}{||\mathbf{x}_{ki}||^2 - \frac{\sum_{j=1}^{M} |\tilde{\mathbf{s}}_{kj}^H \mathbf{x}_{ki}|^2}{N}} \tag{7}$$

Substituting the return signal given by (1) and expanding the above, the dominant terms of the numerator of (7) are

$$\sum_{j=1}^{M} |\alpha_{kji}|^2 N - \sum_{j=1}^{M} |\alpha_{kji}|^2$$

which when divided by the number of signal return samples N, yields an approximation of the numerator of the SDR in (2). The dominant terms of the estimation of the clutter plus interference plus noise power in (7) are the terms $\mathbf{i}^H\mathbf{i}$ and $\mathbf{d}^H\mathbf{d}$, which are the terms in the denominator of (2).

V. SIMULATION RESULTS

First, we verify the validity of (7) using a simulation scenario with two clusters ($K = 2$), each one formed by two radars ($M = 2$). In particular, we examined the accuracy of the estimate of (7) using two different values of the signal return samples N, namely $N = 512$ and 64, when the power used by radars is fixed at $0.1W$. We set the transmitted signals coming from the radars in the same cluster to be orthogonal, while the transmitted signals of radars belonging to different clusters are correlated. The Doppler shift was set to 0.1. The channel gains were randomly chosen as

$$[h_{111}\ h_{121}\ h_{122}\ h_{112}\ h_{211}\ h_{221}\ h_{222}\ h_{212}] = [0.1996\ 0.9917\ 0.3590\ 0.2184\ 0.8920\ 0.6314\ 0.2386\ 0.3794]$$

while the clutter and interference channel gain is given by $\nu_{kji} = h_{kji}/10$ and $\mu_{kji} = h_{kji}/10$, respectively, for all $k, i, j = 1, 2$. The noise power was set to $\sigma_n^2 = 0.01$.

Figures 1 and 2 show both the estimated and the true values of the SDR for the two radars in the first cluster for $N = 512$ and $N = 64$. The results are shown for 50 different random realisations of the signal, clutter and noise. For $N = 512$ we obtain a very good estimate of the SDR, whereas for $N = 64$, the estimated values have larger deviation from the true SDR.

To examine the game theoretic behaviour of our model, we simulated a network of four radars partitioned into two clusters, as before. As the focus is on the convergence to the Nash equilibrium, we run our algorithm for a number of iterations, where each time the transmission power of the radars is updated according to the solution of the optimisation problem (5) until the power is converged. We show the convergence of the power allocation to the equilibrium point for $N = 512$ and $N = 64$ signal return samples, and using the estimation for the SDR as described previously. For the calculation of the target SDR we set $\varepsilon_1 = \varepsilon_2 = 0.05$ in (4). For these values of ε_1 and ε_2, the target γ_k^* for both radars is computed as 2.1599 and 2.1747 for $N = 512$ and $N = 64$, respectively. We also set the minimum and maximum available power to $\underline{p}_{ki} = 0$ and $\overline{p}_{ki} = 1$, respectively, for all four radars in the network.

Figures 3 and 4 depict the SDR throughout the process of the equilibrium convergence of the radars in the first cluster, for the cases where the true and the estimated SDR are for $N = 512$ and 64, accordingly. As the estimation of the SDR is more accurate for $N = 512$, as seen in Figures 5 and 6, convergence to the equilibrium point is more stable with $N = 512$. For $N = 512$, the power allocation in every step of the game theoretic algorithm agrees with that of the power allocation derived using the true value of the SDR, while for $N = 64$, the use of the estimated SDR provides a less

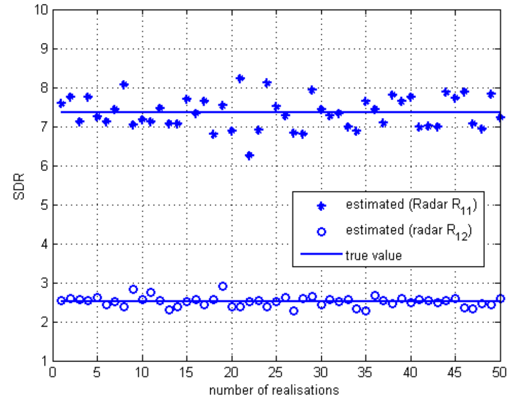

Fig. 1. Estimated and true values of SDR for a fixed transmission power $p_{11} = p_{12} = 0.1W$ and $N = 512$.

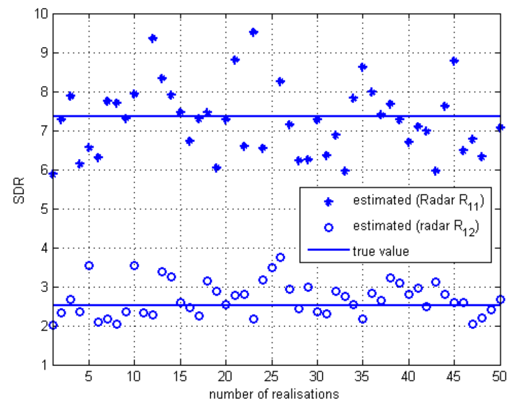

Fig. 2. Estimated and true values of SDR for a fixed transmission power $p_{11} = p_{12} = 0.1W$ and $N = 64$.

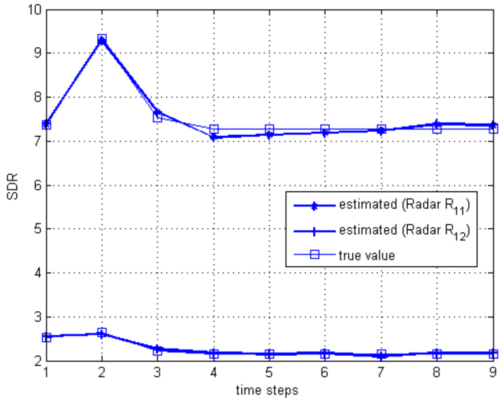

Fig. 3. The SDR values achieved by the game theoretic method using the true value and the estimated value of SDR ($N = 512$) for the radars in the first cluster. The target SDR is 2.1599.

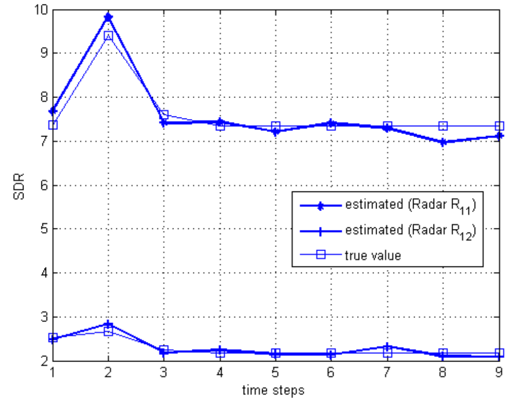

Fig. 4. The SDR values achieved by the game theoretic method using the true value and the estimated value of SDR ($N = 64$) for the radars in the first cluster. The target SDR is 2.1599.

accurate solution. However, for both cases (i.e. $N = 512$ 64), the algorithm has converged to an equilibrium point within 10 iterations.

Table I shows the power allocation and the achieved SDR after the convergence of the game theoretic algorithm. Both the cases corresponding to the use of the true SDR and the estimated value of SDR in the game are shown. For the estimation of the SDR, both $N = 512$ and 64 were considered. The target SDRs for $N = 512$ and 64 were obtained using (4) as 2.1599 and 2.1747. In both the cases, the power allocation and the achieved SDR are very close to that obtained by the game theoretic algorithm that uses the true SDR. Moreover, the results show that only one radar in each cluster is active, while the other radar uses the signal from active radar as signal of opportunity for detection.

To show the generalisation of our scheme regarding the network configuration, we examined the convergence of the game theoretical algorithm for the case of two clusters with three radars per cluster ($K = 2$, $M = 3$). We also considered the case of three clusters with two radars each ($K = 3$, $M = 2$). In both network topologies we compare the convergence of the power to the equilibrium when the true and estimated SDRs are used. The resulting power allocations for the first cluster for the first configuration are shown in Figures 7 and 8, for $N = 512$ and $N = 64$, accordingly. Similarly, the power convergence for the case where the network is formed of three clusters with two radars per cluster, is depicted in Figures 9 and 10 for $N = 512, 64$. As the increase of the number of the signal return samples from $N = 64$ to $N = 512$ leads to more accurate estimation of the SDR, the game theoretic power allocation with the estimated SDR for $N = 512$ is closer to the power allocation values that are obtained using the true value of SDR, for various network configurations.

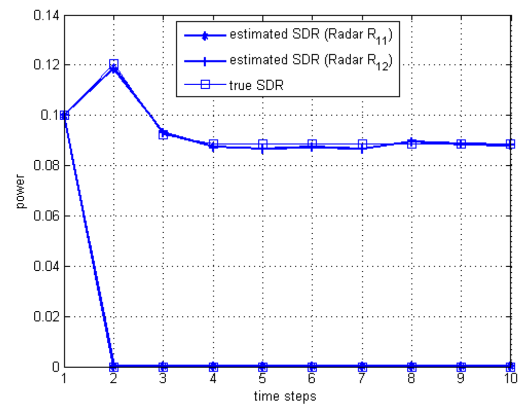

Fig. 5. The power allocated to radars by the game theoretic method using the true value and the estimated value of SDR ($N = 512$) in the first cluster.

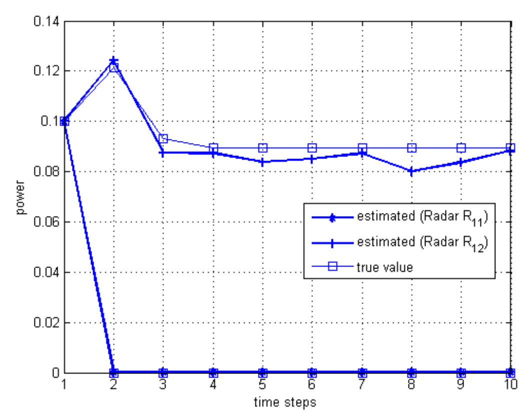

Fig. 6. The power allocated to radars by the game theoretic method using the true value and the estimated value of SDR ($N = 64$) in the first cluster.

VI. Conclusion

We have examined the problem of power allocation in a MIMO radar network within a game theoretic framework, where the radars are grouped into clusters. Our simulation results showed the convergence of the algorithm to the Nash equilibrium of the game. The radars are able to achieve the required SDR values while optimising their transmission

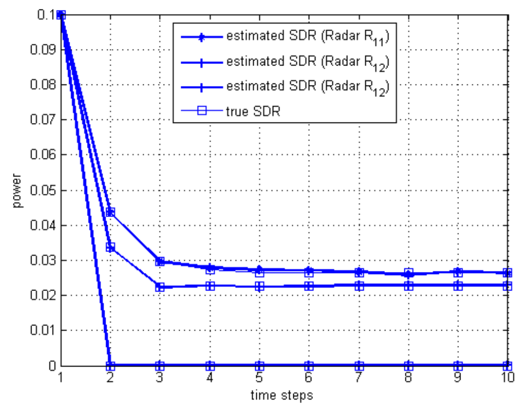

Fig. 7. The power allocated to radars by the game theoretic method using the true value and the estimated value of SDR ($N = 512$) in the first cluster when $K = 2$ and $M = 3$.

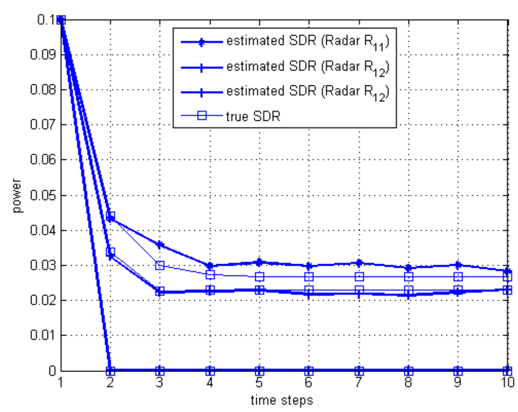

Fig. 8. The power allocated to radars by the game theoretic method using the true value and the estimated value of SDR ($N = 64$) in the first cluster when $K = 2$ and $M = 3$.

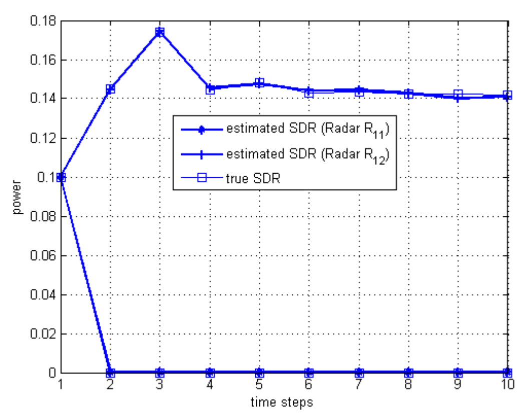

Fig. 9. The power allocated to radars by the game theoretic method using the true value and the estimated value of SDR ($N = 512$) in the first cluster when $K = 3$ and $M = 2$.

power. We also investigated the performance of the game theoretic algorithm in the presence of estimation error and

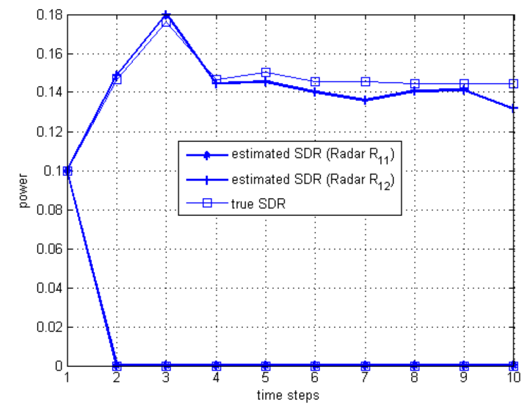

Fig. 10. The power allocated to radars by the game theoretic method using the true value and the estimated value of SDR ($N = 64$) in the first cluster when $K = 3$ and $M = 2$.

TABLE I

THE POWER ALLOCATION AND THE ACHIEVED SDR USING THE GAME THEORETIC METHODS WHEN THE TRUE VALUE AND ESTIMATED VALUE OF THE SDR ARE USED. THE TARGET SDR IS 2.1599 AND 2.1747 FOR N=512 AND N=64 RESPECTIVELY.

$N = 512$ ($\gamma_1^* = \gamma_2^* = 2.1599$)				$N = 64$ ($\gamma_1^* = \gamma_2^* = 2.1747$)			
True		Estimate		True		Estimate	
SDR	Power	SDR	Power	SDR	Power	SDR	Power
7.2793	0	2.2755	0	7.3389	0	6.9271	0
2.1599	0.0886	2.1581	0.0881	2.1747	0.0895	2.0606	0.0882
2.9214	0.0663	2.9310	0.0661	2.9317	0.0668	2.9917	0.0685
2.1599	0	2.1683	0	2.1747	0	2.1921	0

demonstrated that the proposed algorithm has the potential to converge to the equilibrium and achieve the target SDR even in the presence of estimation error. Finally, the game theoretic algorithm was applied to various network topologies and the convergence to the equilibrium was observed.

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