

## On the use of image moments for ATR from SAR images

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### ABSTRACT

*Enhancing target recognition from Synthetic Aperture Radar (SAR) images is a challenging task that cannot be generally solved through a unique and specific sensor configuration or signal processing solution. In particular, solutions exploiting physical target modelling not always are able to deal with complex targets or with small differences between classes. This issue can be solved if image processing techniques are exploited in order to represent the target in a reference domain where small differences and complex structures can have a significant contribution to the target recognition task. The aim of this paper is to provide an overview on the use of image moments for Automatic Target Recognition (ATR) from SAR images. In particular two families of image moments will be considered, pseudo-Zernike and Krawtchouk. Both image moments are computed from orthogonal two-dimensional polynomials that are used as basis to represent the targets' images. The use of image moments introduces advantages in the sense of computational cost, flexibility, reliability and capabilities to identify different targets. Furthermore, these representations can be made rotational, scale and translational invariant, thus allowing operational robustness of algorithms, for example mitigating the lack of image registration between training and test observations. The capabilities of the image moments are discussed together with experimental validation of algorithms. In particular the performance on the MSTAR dataset of military vehicles will be discussed while the Gotcha 3D dataset will be considered for the civilian vehicles case.*

## 1. Introduction

Target recognition of vehicles is a topic of increasing interest and demanding requirements. The knowledge of the vehicles deployed in a specific area of interest is fundamental to understand what kind of threat has to be fought (e.g: Small Intercontinental Ballistic Missile launcher rather than a theatre missile launcher), or to understand the activities in a specific site. Nowadays, of greater interest is to bring the level of knowledge to an identification or characterization stage, where the actual capabilities of the vehicle can be understood based on its equipment. For this reason an Automatic Target Recognition (ATR) algorithm should include the capability to identify small differences among targets, like a specific configuration of a multirole vehicle. Furthermore, the ATR task represents one of the multiple tasks in which modern platforms are involved, for example a UAV (Unmanned Aerial Vehicle) will be acquiring the radar echoes, perform the imaging using High Performance Computing (HPC) capabilities [1], maintain constant communication with a control centre or other platforms, manage other systems like Electro-Optical (EO) sensors. For this reason the processing and the information extraction have to comply with the low Size Weight And Power (SWAP) paradigm. In order to address the identification capability, reliability and low computational cost

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requirements, in this paper we consider the exploitation of image moments for the purpose of ATR from SAR images. In particular the capabilities of target recognition, identification and characterization of two image moments families are investigated, pseudo-Zernike and Krawtchouk. Pseudo-Zernike moments belong to the family of geometric moments such as Hu and Zernike moments [2], [3], which were used both in image processing for pattern recognition and image reconstruction [4,5,6]. Some of the main advantages of these moments include position, scale, and rotational invariance. Another important property is that pseudo-Zernike (pZ) are independent moments, because they are computed from orthogonal polynomials. A common issue to most of families of image moments is represented by the discretization error and poor robustness in low Signal to Noise Ratio (SNR) conditions [3]. This error builds up as the order increases, limiting the accuracy of the computed moments. This drawback would translate in target recognition algorithms with less accuracy in discriminating between targets that differ in small components, that would be picked up by accurate higher order moments. This issue is addressed by using Krawtchouk (Kr) moments, which are characterized by some peculiar properties [7], in particular they are discretely defined, thus there is no requirement of spatial normalization and the discretization error is nonexistent. This translates in a relaxation on the amount of resource required to represent and store the polynomials.

The computational cost is reduced thanks to the orthogonality property of pZ and Kr polynomials that relaxes the requirements of feature selection to mitigate overfitting.

These characteristics, together with the capability to pre-compute the polynomials, make image moments compatible with SWAP systems.

The remainder of the paper is organized as follows, Section 2 introduces pZ and Kr moments demonstrating translational and rotational invariance, useful in the ATR from SAR application, and then the ATR algorithm is described. Section 3 presents the results obtained using the MSTAR and Gotcha dataset, Section 4 concludes the paper.

## 2. Pseudo-Zernike and Krawtchouk Moments

In this section the two families of image moments exploited to represent the targets present within the SAR image are described. The invariant properties with respect to translation and rotation are also shown in this section as important characteristics for the specific SAR based ATR application.

### 2.1. Pseudo-Zernike Moments

Let  $f(x, y)$  be a non-negative real image. The complex pseudo-Zernike moments [8] can be computed as

$$\begin{aligned} \psi_{n,l} &= \frac{n+1}{\pi} \int_0^{2\pi} \int_0^1 W_{n,l}^*(\rho \cos \theta, \rho \sin \theta, \rho) f(\rho \cos \theta, \rho \sin \theta) \rho d\rho d\theta, \end{aligned} \quad (1)$$

where the symbol  $(\cdot)^*$  indicates the complex conjugate operator and  $W_{n,l}$  are the pseudo-Zernike polynomials. The latter are a set of orthogonal functions that can be written in the form

$$W_{n,l}(x, y, \rho) = W_{n,l}(\rho \cos \theta, \rho \sin \theta, \rho) = S_{n,l}(\rho) e^{il\theta}, \quad (2)$$

with  $i = \sqrt{-1}$ ,  $x = \rho \cos \theta$ ,  $y = \rho \sin \theta$ ,  $l$  an integer, and  $S_{n,l}(\rho)$  a polynomial (called radial polynomial) in  $\rho$  of degree  $n$  such that  $n \geq |l|$ . Notice that the modulus of (2) is rotationally invariant [8]. Moreover,

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these functions form a complete basis and satisfy, on the unit disc (i.e. for  $x^2 + y^2 \leq 1$ ), the orthogonality relation [8]

$$\begin{aligned} \iint_{x^2+y^2 \leq 1} W_{n,l}^*(x, y, \sqrt{x^2 + y^2}) W_{m,k}(x, y, \sqrt{x^2 + y^2}) dx dy \\ = \frac{\pi}{n+1} \delta_{mn} \delta_{kl}, \end{aligned} \quad (3)$$

where  $\delta_{mn}$  is the Kronecker delta function, i.e.  $\delta_{mn} = 1$  if  $m = n$ , and 0 otherwise. As given in [8], an explicit expression to compute the radial polynomials,  $S_{n,l}(\rho)$ , is

$$S_{n,l}(\rho) = \sum_{k=0}^{n-|l|} \frac{\rho^{n-k} (-1)^k (2n+1-k)!}{k! (n+|l|+1-k)! (n-|l|-k)!}. \quad (4)$$

These moments have been widely used in many applications, such as image reconstruction and classification thanks to their orthogonality and low computational cost. Moreover, as previously stated an important characteristic of the pseudo-Zernike moments is the simple rotational transformation property due to (2); indeed, the moment requires only a phase factor for the rotation [8].

## 2.2. Krawtchouk Moments

Multiple versions of the Krawtchouk moments have been defined [7], and those that for their characteristics fit better the final goal of a target recognition algorithm are those computed using the weighted Krawtchouk polynomials [7]. These polynomials can be obtained starting from the classical formulation, introduced in [9], in the form of a set of orthogonal polynomials associated with the binomial distribution. The classical Krawtchouk polynomials of order  $n$  are defined as [9]

$$K_n(x; p, N) = \sum_{k=0}^N a_{k,n,p} x^k = {}_2F_1\left(-n, -x; -N; \frac{1}{p}\right) \quad (5)$$

where  $x$  and  $n$  belong to  $(0, 1, 2, \dots, N)$ ,  $N > 0$ ,  $N \in \mathbb{N}$ , where  $\mathbb{N}$  is the set of natural numbers,  $p$  is a real number belonging to the set  $(0, 1)$ , and  ${}_2F_1$  is the Gauss hypergeometric function

$${}_2F_1(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{(c)_k k!} \quad (6)$$

where  $(\cdot)_k$  is the Pochhammer symbol given by

$$(a)_k = a(a+1) \dots (a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} \quad (7)$$

The polynomials in (5) suffer of numerical stability and to overcome this limitation a weight is required [7], leading to its weighted form, i.e.

$$\bar{K}_n(x; p, N) = K_n(x; p, N) \sqrt{\frac{w(x; p, N)}{\rho(n; p, N)}} \quad (8)$$

with  $w(x; p, N) = \binom{N}{x} p^x (1-p)^{N-x}$  and  $\rho(n; p, N) = (-1)^n \left(\frac{1-p}{p}\right)^n \frac{n!}{(-N)_n}$ .

The polynomials defined in (8) are orthogonal, as the following condition holds

$$\sum_{x=0}^N \bar{K}_n(x; p, N) \bar{K}_m(x; p, N) = \delta_{nm}, \quad \forall p, N \quad (9)$$

Furthermore, the parameter  $p$  represents a shift parameter that can thus be exploited if the focus is on a specific area of interest. Considering a 2D function of interest  $f(x, y)$ , e.g. the modulus of a SAR image, with  $x$  and  $y$  natural numbers belonging, respectively, to the sets  $(1, N)$  and  $(1, M)$ , respectively, and  $M$  and

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$N$  representing the image width and height in samples, the Krawtchouk moment of order  $(n, m)$  is defined as

$$Q_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \bar{K}_n(y; p_1, N-1) \bar{K}_m(x; p_2, M-1) f(x, y) \quad (10)$$

As for the pseudo-Zernike moments also Krawtchouk can be invariant for translation, rotation and scale, and these properties can be exploited at convenience depending on the application.

### 2.3. Invariant Properties

For the specific application of ATR from SAR images two properties of both pZ and Kr moments can result very useful: translational and rotational invariance. These two properties can be exploited in the general case in which the actual position and orientation of the target in the image plane is not known. To demonstrate this capability we consider the pZ moments calculated on two different targets drawn from the MSTAR database and whose position in the image plane is translated and rotated.

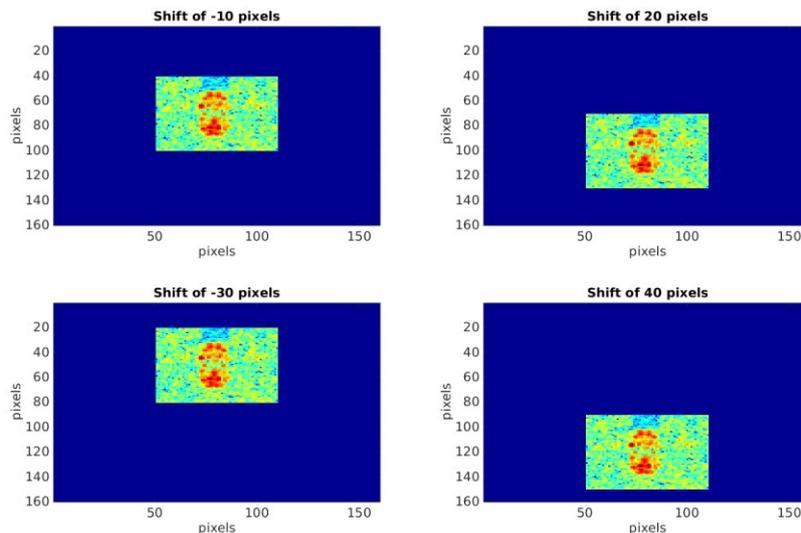


Figure 1: Target 1 translated in the image of -10, 20, -30 and 40 pixels.

In Figure 1 four realizations of Target 1 shifted by -10, 20, -30 and 40 pixels are shown, the whole 160x160 pixels image is considered in the computation of the pZ moments. The same shifted realizations have been also considered for Target 2 and are shown in Figure 2.

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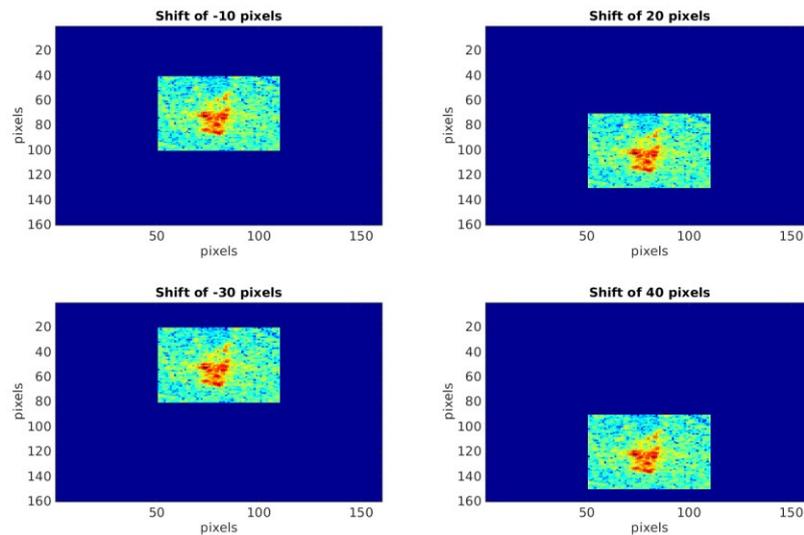


Figure 2: Target 2 translated in the image of -10, 20, -30 and 40 pixels.

By computing the translation invariant pZ moments over the shifted images for both Targets 1 and 2, the feature vectors result to be the same, as shown in Figure 3, where the plot of the natural logarithm of the features is reported for the 4 shifts for moments up to order 5 (36 features).

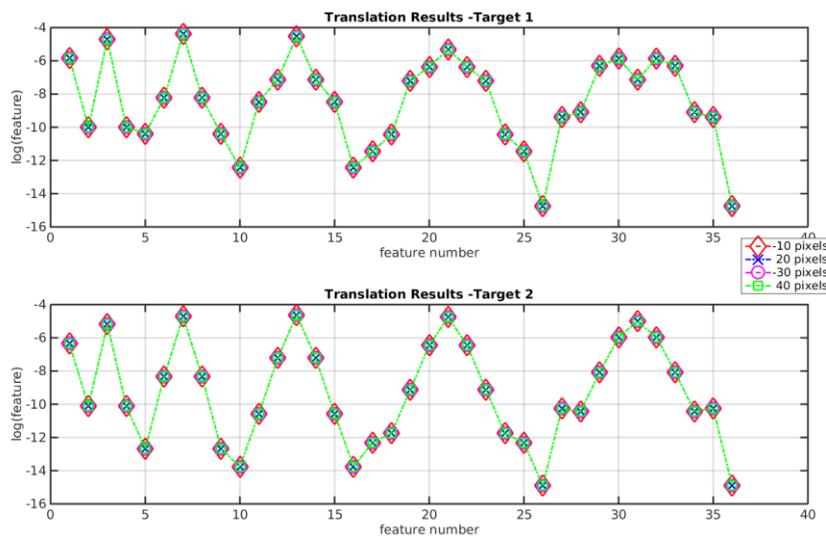
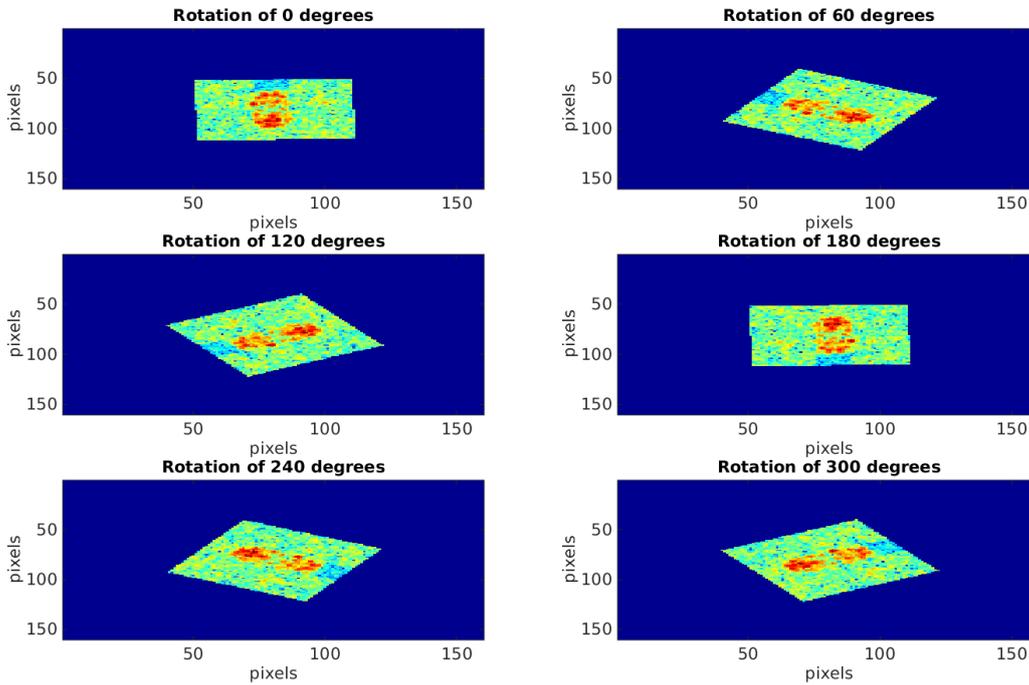


Figure 3: Results obtained in the translation experiment for Target 1 (top) and Target 2 (bottom).

From the results in Figure 3 it can be appreciated how the pZ moments maintain the invariance with respect to the translation of the target, in particular it can be seen how all the markers representing the feature values overlap.

A similar analysis is performed to show the rotation invariance, the images of both targets are rotated of 0, 60, 120, 240 and 300 degrees before the computation of the pZ moments, the analysed cases for Targets 1 and 2 are shown in Figures 4 and 5, respectively.

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**Figure 4: Target 1 rotated of 0, 60, 120, 180, 240 and 300 degrees.**

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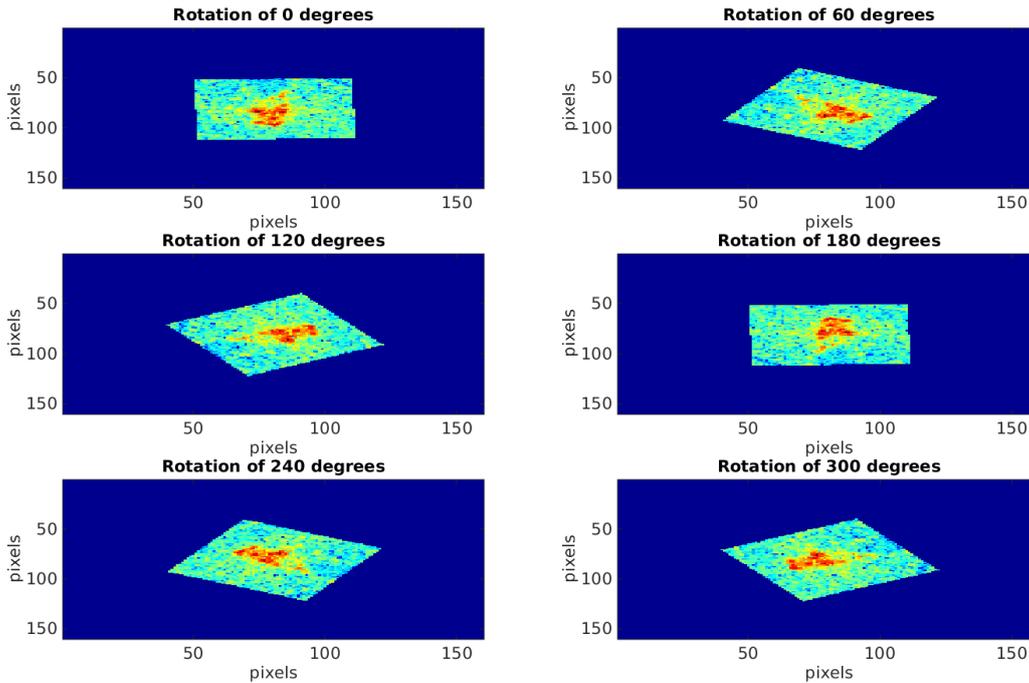


Figure 5: Target 2 rotated of 0, 60, 120, 180, 240 and 300 degrees.

The moments computed up to order 5 for various rotation angles of the original target images are shown in Figure 6, where it the differences between the different realizations of the same feature result to be minimal. This behaviour is mainly due to the numerical error injected by the interpolation process required in the generation of the rotated images before the moments calculation. Also in this case the maximum variance among the different features was computed and it resulted to be  $8.84 \times 10^{-8}$  and  $2.65 \times 10^{-7}$  for Target 1 and 2, respectively.

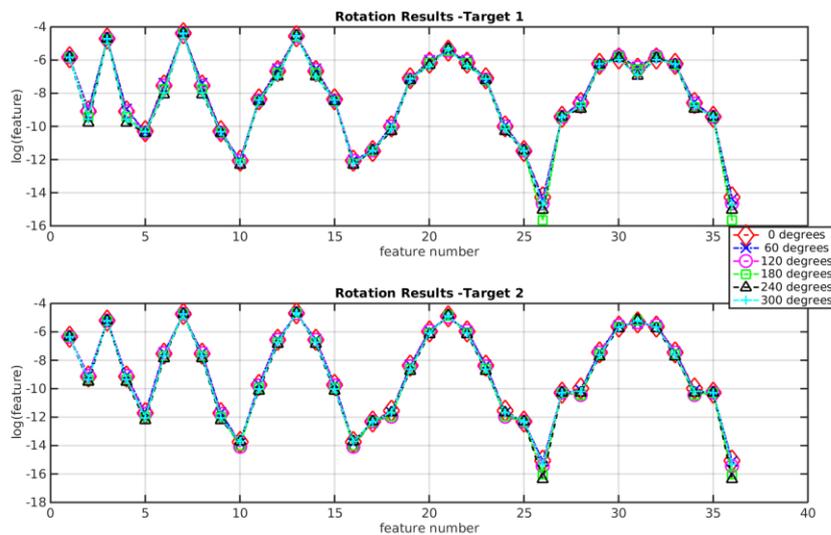
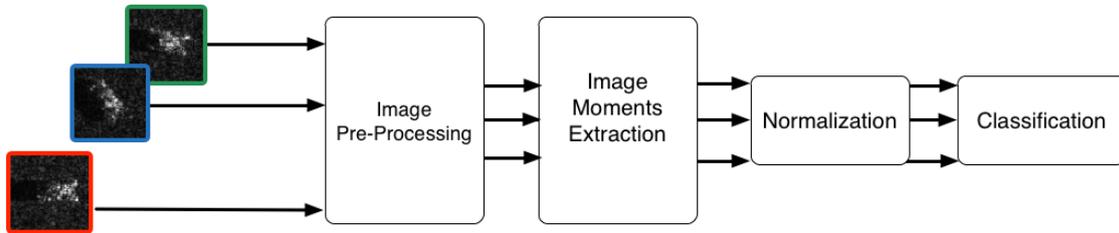


Figure 6: Results obtained in the rotation experiment for Target 1 (top) and Target 2 (bottom).

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**2.4. Algorithm Description**

The functional blocks of the proposed ATR algorithm exploiting image moments are depicted in Figure 7.



**Figure 7: Functional block of the image moments based ATR algorithm.**

The starting point is the SAR image containing one of the possible targets of interest. Depending on the specific sensor used to acquire the images, a pre-processing consisting of a data transformation might be required. For example in the Gotcha [9] dataset it is required to reduce the image dynamic range through a log transformation.

By applying equation (1) or (10), then  $pZ$  or  $Kr$  for a given order can be computed. The moments computed up to a given order are used to populate a feature vector that is then statistically normalized in order to avoid feature domination phenomena [11].

The feature vectors are then used as input to a classification algorithm, such as  $k$ -Nearest Neighbours ( $k$ -NN), Support Vector Machine, or Bayesian Classifier. The output of the classifier contains the class identification index for each image under test.

**3. Performance Analysis on the MSTAR dataset**

In this section the performance analysis of the proposed algorithm are assessed on real data. The MSTAR dataset is a collection of SAR images of 14 different military targets [12], that represents an invaluable test-bench for ATR algorithms. This dataset can be used for the different levels of target classification. According to the NATO AAP-6 Glossary Terms and Definitions, with “recognition” is meant the classification of the type/category of target; “identification” regards the capability to assign the target to a subclass; “characterization” takes into account the class variants. Following these definitions, Table 1 reports the different targets and their grouping in the MSTAR dataset.

**Table 1: List of targets in the MSTAR dataset and their grouping in different sets for Recognition, Identification and Characterization.**

| Description | Target    | Recognition | Identification | Characterization |
|-------------|-----------|-------------|----------------|------------------|
| Tank        | BMP2 9563 | A1          | B1             | C1               |
| Tank        | BMP2 9566 |             |                | C2               |
| Tank        | BMP2 C21  |             |                | C3               |
| Tank        | T72 132   |             | B2             | C4               |
| Tank        | T72 812   |             |                | C5               |

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|                   |           |    |     |     |
|-------------------|-----------|----|-----|-----|
| Tank              | T72 S7    |    |     | C6  |
| Tank              | 2S1       |    | B3  | C7  |
| Tank              | T62       |    | B4  | C8  |
| Tank              | ZSU       |    | B5  | C9  |
| Personnel Carrier | BTR70 C71 | A2 | B6  | C10 |
| Personnel Carrier | BTR60     |    | B7  | C11 |
| Truck             | ZIL131    | A3 | B8  | C12 |
| Reconn. Vehicle   | BRDM      | A4 | B9  | C13 |
| Bulldozer         | D7        | A5 | B10 | C14 |

A similar target grouping can be also performed for the targets present in the Gotcha 3D volumetric SAR dataset [9] with 720 available polarimetric SAR images (obtained by focussing the data considering 4 degrees in azimuth aperture as in [13]). This dataset contains 9 civilian vehicles of different sizes that can be used to group these in different subclasses as shown in Table 2.

**Table 2: List of targets in the Gotcha dataset and their grouping in different sets for Recognition, Identification and Characterization.**

| Description      | Target               | Recognition | Identification | Characterization |
|------------------|----------------------|-------------|----------------|------------------|
| Compact Car      | E – Nissan Sentra    | A1          | B1             | C1               |
| Compact Car      | J – Chevy Prizm      |             |                | C2               |
| Mid-Size Car     | A – Chevy Malibu     |             | B2             | C3               |
| Mid-Size Car     | B – Toyota Camry     |             |                | C4               |
| Full-Size Car    | C – Ford Taurus      |             | B3             | C5               |
| Full-Size Car    | D – Nissan Maxima    |             |                | C6               |
| SUV              | F – Hyundai Santa Fe |             | B4             | C7               |
| Industrial Truck | C1 – Case Tractor    | A2          | B5             | C8               |
| Industrial Truck | C2 – Fork Lift       |             |                | C9               |

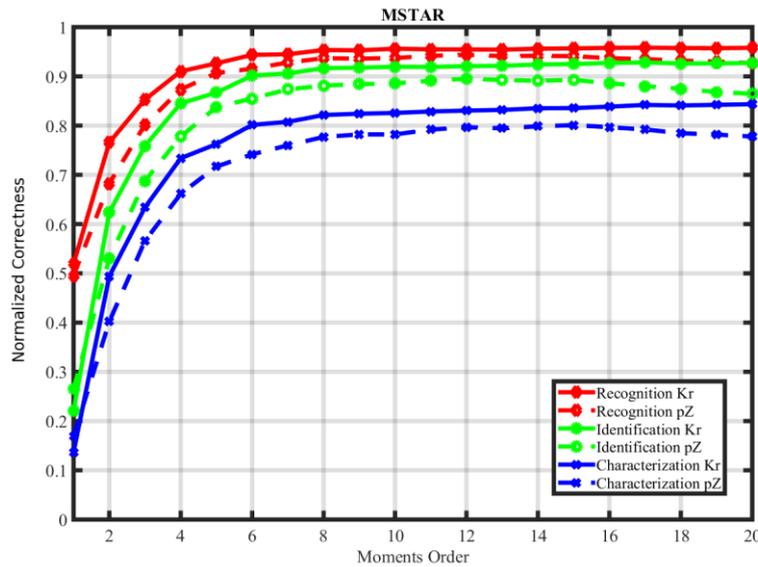
The performance obtained with both pZ and Kr is reported in Figure 8 and Figure 9, for the MSTAR and Gotcha dataset, respectively. These results are obtained averaging the performance of 100 Monte Carlo runs with random selection of the training and test sets as input to a *k*-NN classifier.

For the Gotcha dataset, the training set is formed by all the images coming from the lowest altitude of 9 different passes (each of them with different altitude, thus different grazing angle), which were carried out

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to collect the SAR images. The same is done for the MSTAR dataset, for which only two passes are available.

The parameter  $k$  of the classifier is 3. In order to deal with the uncertainty of the decision, if the maximum output score of the classifier is not unique, the target under test is assigned to the nearest class (that is, a 1-NN is used).



**Figure 8: Performance in terms of normalized correct recognition, identification and characterization for pseudo-Zernike and Krawtchouk moments on the MSTAR dataset.**

Figure 8 reports the results obtained on the MSTAR dataset showing a high level of correct recognition, identification and characterization for both pZ and Kr approaches. However, considering moments up to order 10, the latter approach results to provide better performance with 95.52%, 91.95% and 82.57% of correct target recognition, identification and characterization while pZ performance were equal to 93.73%, 88.56% and 78.21% for the same levels of target discrimination. The fact that when using Kr the performance in characterization increases significantly, suggests that this approach has higher capabilities in representing different variations of the same vehicle.

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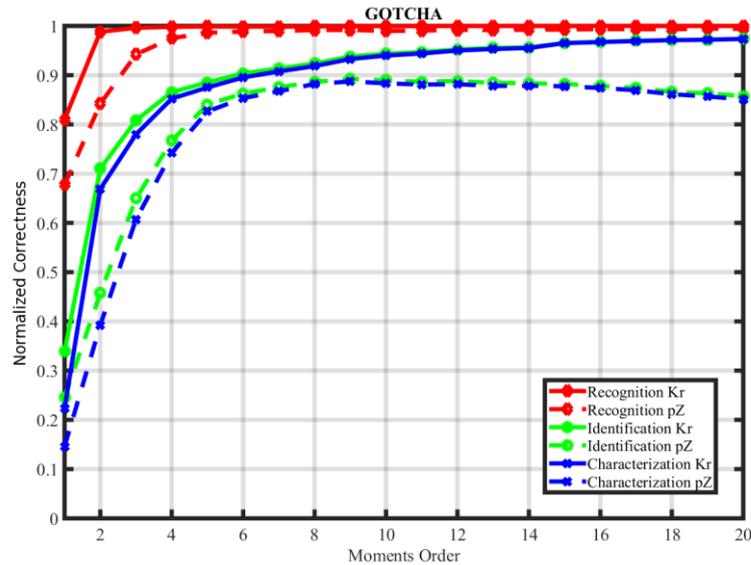


Figure 9: Performance in terms of normalized correct recognition, identification and characterization for pseudo-Zernike and Krawtchouk moments on the Gotcha dataset.

Considering the results shown in Figure 9, where the performance obtained from the Gotcha dataset with civilian vehicles is reported, it is evident that a high level of correctness can be achieved with both pZ and Kr even when only few moments are considered. However, the superiority of Kr with respect to pZ is confirmed also in this case, considering again moments up to order 10, with 99.60%, 94.36% and 93.39% for correct target recognition, identification and characterization versus 98.98%, 88.98%, 88.34% obtained using pZ. Also in this case finer the target details are required, better Kr performs compared to pZ, confirming the observation made for the MSTAR case.

This behaviour is principally due to the nature of the basis functions used to compute the Kr moments that are discrete defined and are therefore immune to discretization error that can affect the capability to represent finer details.

## 4. Conclusions

The aim of this paper has been to provide an overview on the use of image moments for ATR from SAR images. In particular two families of image moments have been considered, pseudo-Zernike and Krawtchouk. Both image moments are computed from orthogonal two-dimensional polynomials that are used as basis to represent the targets' images. An algorithm exploiting these moments to perform ATR has been introduced and the performance in terms of target recognition, identification and characterization were assessed for military and civilian vehicles. Overall, both approaches result to provide high ATR capabilities for both families of vehicles with superior performance obtainable when Krawtchouk moments are exploited, in particular for the target characterization task.

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